# STABILIZATION OF CHAOS RIKITAKE SYSTEM BY USE OF FUZZY CONTROLLER

**[1]Yousof Gholipour, [2,3]Mahmood Mola**

[1]Department of Electrical Eng., Mehriz Branch. Islamic Azad University. Mehriz.Iran
[2]Department of Electrical Eng., Science and Research Branch. Islamic Azad University. Tehran, Iran.
[3]Department of Electrical Eng., University of A. A. Boroujerdi, Boroujerd, Iran.
Contact: TEL: +983525229100; FAX: +983525229102; y.gholipour@yahoo.com

***ABSTRACT*****;** *In this paper, controlling of the chaos Rikitake system has been investigated. To this end, we used the fuzzy controller Takagi–Sugeno (T-S) method, that applies to chaotic systems. In the first instance, we monitored behavior of Rikitake system, after that we designed a controller base on fuzzy T-S method, then applied it to the chaos Rikitake system and monitor the behavior of the system. Duration the paper, Numerical simulations are given to illustrate the effectiveness and validity of the proposed approach.*

**Keywords;** Chaotic systems, Rikitake system, fuzzy controller, Takagi–Sugeno model.

## INTRODUCTION

Many theories have been advanced to explain the origin of the earth's main dipole field, the Rikitake system introduced for describing the irregular polarity switching of the Earth's magnetic field. But intervals among such geomagnetic polarity reversals are highly irregular. Thus, while their average is about $3\times10^5$ years, there are intervals as long as $3\times10^7$ years without polarity change. From introduce Rikitake system, till now many control approaches have been presented. At the bellow we will review this works together:

Pecorra and Carroll [1], Ott.E, *et.al*.[2], Carlos Aguilar-Ibañez, et.al. [3], Mohammad Ali Khan [4] Synchronization for chaotic system has been investigated. In the last years, some methods to achieve synchronization have been proposed from the control theory perspective, such as the famous observer-based approach [5], [6], and the so-called adaptive synchronization method [7]. Two research directions have been already conformed in synchronizing chaos: (i) analysis and (ii) synthesis. Analysis problem comprises: (a) the classification of synchronization phenomena [8], (b) the construction of a general framework for unifying chaotic synchronization [9], and (c) the comprehension of the synchronization properties, for instance, robustness [10] or geometry [11]. Liu Xiao-Jun, *et.al*. [12] analyzed the dynamics of Rikitake two-disk dynamo to explain the reversals of the Earth's magnetic field. They concluded that the chaotic behavior of the system can be used to simulate the reversals of the geomagnetic field. The Rikitake chaotic attractor was studied by several authors. T. McMillen [13] and Mohammad Javidi *et.al*.[14] has studied the shape and dynamics of the Rikitake attractor. J. Llibre .*et.al* [15] used the Poincare compactification to study the dynamics of the Rikitake system at infinity. Chien-Chih Chen *et.al* [16] have studied the stochastic resonance in the periodically forced Rikitake dynamo. In the past decade, many researchers start working on controlling the chaotic behaviors. Harb and Harb [17] have designed a nonlinear controller to control the chaotic behavior in the phase-locked loop by means of nonlinear control. Ahmad Harb[18] have designed a controller to control the unstable chaotic oscillations by means of back stepping method. U.E. Vincent, R. Guo [19], Park *et.al* [20] and Jeong *et.al*[21] They have presented a controller by use of adaptive method and controlled chaotic Rikitake system.

In this paper we want to control chaotic Rikitake system by use of Takagi–Sugeno (T-S) fuzzy model [22] has attracted a great deal of attention. The main purpose of the T-S fuzzy model is to represent or approximate a complex nonlinear system. The T-S fuzzy model approach will provide a powerful method for analysis of nonlinear systems [23, 24]. In other words fuzzy control has been proved to be a powerful method for the control problem of complex nonlinear systems. For many real life systems, which are highly complex and inherently nonlinear, conventional approaches to modeling often cannot be applied whereas the fuzzy approach might be the only appropriate alternative [28]. Takagi-Sugeno fuzzy model and the so-called parallel distributed compensation, a controller structure devised in accordance with the fuzzy model [29].We will write state space of Rikitake system after that write fuzzy rules T-S method and obtain matrix of new state space, later rewrite equation while will apply a control signal to the input signal and monitor the effect of fuzzy controller on the chaotic Rikitake system.

The remaining of this work is organized as follows. In Section 2 we will describe mathematical of Rikitake system and will monitor the behavior of system in a usual situation, without a controller. In section 3 the modeling of the Rikitake system has been presented, that for more convenience, the equations of the system will be written in the state-space matrix form and on base of this form will continue and write fuzzy rules and calculate control signal. In the next section will apply control signal on chaotic system and while, will assume the initial conditions and will monitor the effect of our controller on this chaotic system. At the end conclusion has been presented.

**Description mathematical of Rikitake system**

The original differential equations derived by Rikitake are:

$$\begin{cases} L_1 \dfrac{dI_1}{dt} + R_1 I_1 = \omega_1 M I_2 \\ L_2 \dfrac{dI_2}{dt} + R_2 I_2 = \omega_2 M I_1 \\ C_1 \dfrac{d\omega_1}{dt} = G_1 - M I_1 I_2 \\ C_2 \dfrac{d\omega_2}{dt} = G_2 - N I_1 I_2 \end{cases} \tag{1}$$

where L, R are the self-inductance and resistance of the coil, the electric currents, I, $\omega$, C, G are the electric currents, the angular velocity , momentum of inertia, and the driving force; M, N are the mutual inductance between the coils and the disks.
Now we consider a further simplification by
$L_1=L_2$, $R_1=R_2$, $M=N$, $C_1=C_2$, $G_1=G_2$
and set:

$$I_1 = \sqrt{\frac{G}{M}}x, \quad I_2 = \sqrt{\frac{G}{M}}y, \quad \omega_1 = \sqrt{\frac{GL}{CM}}z \tag{2}$$

$$\omega_2 = \sqrt{\frac{GM}{CM}}(z-a), \quad t = \sqrt{\frac{CL}{GM}}t', \quad u = R\sqrt{\frac{C}{LGM}} \tag{3}$$

Where, constant parameter a,u>0.
The system mathematical model can be written as follows:

$$\begin{cases} x_1' = -ux_1 + x_2 x_3 \\ x_2' = -ux_2 + (x_3 - a)x_1 \\ x_3' = 1 - x_1 x_2 \end{cases} \tag{4}$$

Where $(x_1, x_2, x_3) \in R3$ are the state variables and a > 0, u > 0 are parameters. Note that system (4) is a quadratic system in $R^3$. The choice of the parameters a > 0 and u > 0 reflects a physical meaning in the Rikitake model. For study physical meaning can see [30]. Here we suppose a=5 & u=2, we know according to ref. [32] this system in some values is unstable and we choose this system in chaos mode.
Note that x and y are corresponding to the electric currents, while z is corresponding to the angular velocity. For more details read ref. [30, 31].

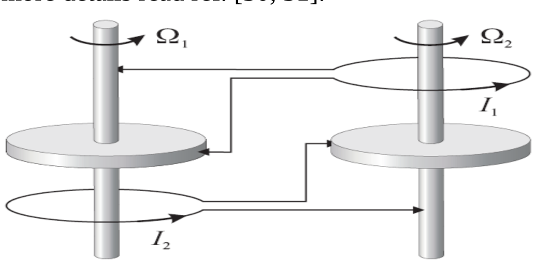


**Fig.1 The Rikitake dynamo is composed of two disk dynamos coupled to another**

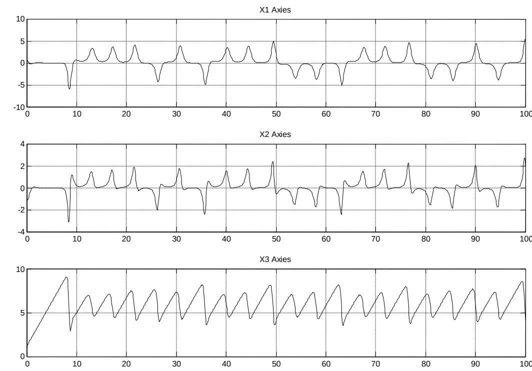


**Fig.2. Behavior of Rikitake system without controller, in t=100 seconds**

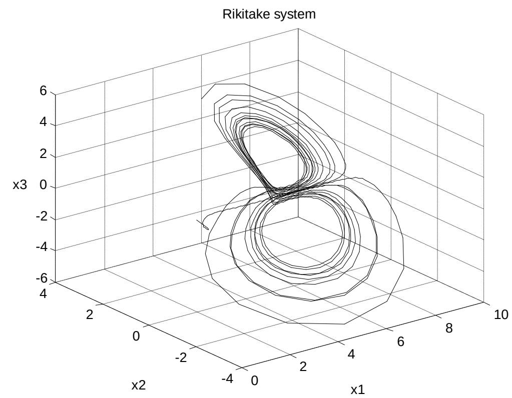


**Fig.3. Behavior of Rikitake system in 3D plot**

## Modeling of the Rikitake system

To realize a fuzzy model-based design with parametric uncertainty, chaotic systems should first be represented by fuzzy models. For this point we, at the first rewrite equations of Rikitake system on base of state space after that obtain fuzzy model.
For more convenience, the equations of system can be written in the state-space matrix form as:

$$\dot{x}(t) = Ax(t) + B \tag{5}$$

where $A \in R^{n\times n}$, $B \in R^{n\times n}$ are constant matrix, therefore we will have:

$$A = \begin{bmatrix} -u & x_3(t) & 0 \\ x_3(t)-a & -u & 0 \\ 0 & -x_1(t) & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^{\mathrm{T}} \in R^3$$

And we suppose that $Z_1(t)=X_3(t)$ and $Z_2(t)=X_1(t)$ then we can write:

$$A = \begin{bmatrix} -u & z_1(t) & 0 \\ z_1(t)-a & -u & 0 \\ 0 & -z_2(t) & 0 \end{bmatrix}$$

$$x(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^{\mathrm{T}} \in R^3$$

Now we want to calculate the values of $Z_1(t)$, $Z_2(t)$ when $X_1 \in [1\ 3]$ and $X_3 \in [1\ 3]$ then we will get following values:

$$\underset{x_1,x_3}{Z_1} = \begin{cases} 3 & \max \\ 1 & \min \end{cases}$$

$$\underset{x_1,x_3}{Z_2} = \begin{cases} 3 & \max \\ 1 & \min \end{cases}$$

From min and max values of $Z_1(t)$, $Z_2(t)$ we can write:

$$z_1(t) = x_3(t) = M_{11}(z_1(t)) \cdot (3) + M_{12}(z_1(t)) \cdot (1)$$

$$z_2(t) = x_1(t) = M_{21}(z_2(t)) \cdot (3) + M_{22}(z_2(t)) \cdot (1)$$

Where:

$$M_{11}(z_1(t)) + M_{12}(z_1(t)) = 3$$

$$M_{21}(z_2(t)) + M_{22}(z_2(t)) = 3$$

Therefore, we can calculate the membership function as bellow:

$$M_{11}(z_1(t)) = \frac{1}{2}(x_3(t) - 3)$$

$$M_{12}(z_1(t)) = \frac{1}{2}(9 - x_3(t))$$

$$M_{21}(z_2(t)) = \frac{1}{2}(x_1(t) - 3)$$

$$M_{22}(z_2(t)) = \frac{1}{2}(9 - x_1(t))$$

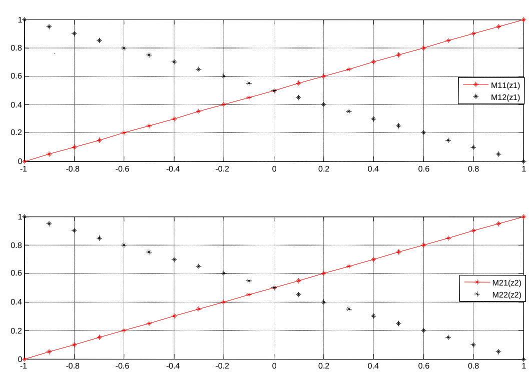


**Fig.4. Membership functions $M_{11}$, $M_{12}$, $M_{21}$ and $M_{22}$**

From $M_{11}$, $M_{12}$, $M_{21}$ and $M_{22}$ we want to write rules of T-S fuzzy models:

Rule 1: *If $Z_1(t)$ is $M_{11}$ and $Z_2(t)$ is $M_{21}$* then:

$$\dot{x}(t) = A_1 x(t) + B + Cu(t)$$

Rule 2: *If $Z_1(t)$ is $M_{11}$ and $Z_2(t)$ is $M_{22}$* then:

$$\dot{x}(t) = A_2 x(t) + B + Cu(t)$$

Rule 3: *If $Z_1(t)$ is $M_{12}$ and $Z_2(t)$ is $M_{21}$* then:

$$\dot{x}(t) = A_3 x(t) + B + Cu(t)$$

Rule 4: *If $Z_1(t)$ is $M_{12}$ and $Z_2(t)$ is $M_{22}$* then:

$$\dot{x}(t) = A_4 x(t) + B + Cu(t)$$

From these rules will obtain:

$$A_1 = \begin{bmatrix} -u & 1 & 0 \\ 1-a & -u & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} -u & 1 & 0 \\ 1-a & -u & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -u & -1 & 0 \\ -1-a & -u & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad A_4 = \begin{bmatrix} -u & -1 & 0 \\ -1-a & -u & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We suppose that:

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D = [0]$$

By use of defuzzifier method we can reformulate the Rikitake system:

$$\dot{x}(t) = \sum_{i=1}^{4} h_i(z(t)) \{A_i x(t) + B\} \qquad (6)$$

And

$$h_1(z(t)) = M_{11}(z_1(t)) \times M_{21}(z_2(t))$$

$$h_2(z(t)) = M_{11}(z_1(t)) \times M_{22}(z_2(t))$$

$$h_3(z(t)) = M_{12}(z_1(t)) \times M_{21}(z_2(t))$$

$$h_4(z(t)) = M_{12}(z_1(t)) \times M_{22}(z_2(t))$$

We can simplify and write the controlled system bellow:

$$\dot{y}(t) = \sum_{i=1}^{4} h_i(z(t)) \{A_i y(t) + B + C\ u(t)\} + D \qquad (7)$$

Where

$e(t) = [y_1 - x_1 \quad y_2 - x_2 \quad y_3 - x_3]^T \in R^3$ and C is the constant matrix.

We have:

$u(t) = [u_1 \quad u_2 \quad u_3]^T$ That it is the control input, and it's equals to $u = C \times e$ .

## RESULTS AND DISCUSSION

In this part to verify and in order to test the effectiveness and feasibility of the presented fuzzy control method, the simulation results have been performed.

The initial conditions of the drive and response system are chosen to be $(x_1, x_2, x_3) = (1, -1, 1)$ and $(y_1, y_2, y_3) = (2, -2, 2)$ respectively.

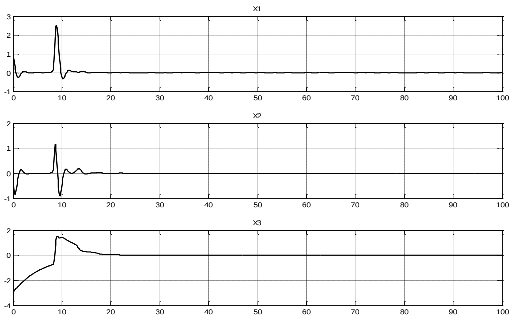


**Fig.5. Behavior of Rikitake system after control, in t=100 seconds**

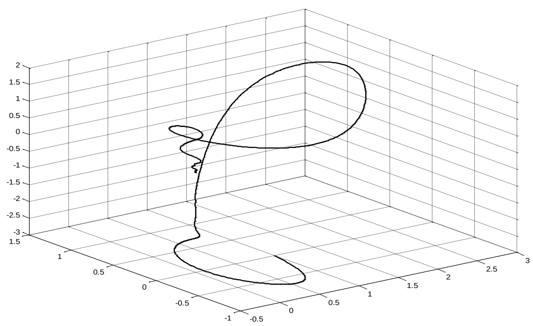

**Fig.6. Behavior of Rikitake system after control in 3D plot**

## CONCLUSION

In this paper, we investigated chaotic Rikitake system and monitored the behavior of the system. We designed and suggested, on base of fuzzy logic T-S(Takagi–Sugeno fuzzy control techniques), a controller and applied this fuzzy controller on chaotic system and monitored behavior of the system. We observed the speed and effect of fuzzy controller on this system, In Comparison to other controllers, this controller works faster and more accurate than other controllers. Finally, we conclude from numerical simulation results that fuzzy controller is more effective than similar controllers.

## ACKNOWLEDGMENT

We acknowledge our friend, Mehdi fatemi and the Associate Editor and anonymous reviewers for their valuable comments and suggestions that have helped us to improving the paper.

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