AN APPLICATION OF INTERVAL VALUED FUZZY SOFT MATRIX (IVFSM) IN DECISION MAKING

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ABSTRACT: *In this era of high technology and scientific advancement, the complexities and oddities of life seek its solutions and comforts from various fields of mathematics. One of these helpful hands of mathematics is soft set theory. In this paper, we have proposed IVFSM, its kinds with meaningful illustrations and have proved its utility in decision making problems. Moreover, its distinctive properties have also been highlighted.*

Keywords: Soft set, fuzzy soft set, interval valued fuzzy set, interval valued soft set , soft matrix, interval valued fuzzy soft set

INTRODUCTION

Today, life has become a real bed of thrones than ever before. Zadeh's classical concept of fuzzy set [1] is hard to cope with such emerging issues. From the beginning of fuzzy set theory and its higher order fuzzy sets, there erupted many opinions about its feasibility and applicability. Despite controversial nature of fuzzy sets, Atanassov, introduced Intuitionistic fuzzy set [2] which proved to be extremely handy and applicable.

Since the very inception of Soft set theory proposed by Molodtsov [3] it met with high appreciation. After that, Maji et al. [4] presented a new theory of SS. Maji et al is also remembered for its curious milestone he got by combining both Intuitionistic fuzzy set and SS resulting in Intuitionistic fuzzy soft set [5]. Majumdar et al. [6] also worked upon generalized FSS.

IVFSS is combination of IVFS and soft sets, which is the consequence of Yang et al. [7] and algorithm is also coined to reach the bottom of IVFSS. The biggest achievement attained by Sarala and Prabhavathi [8], use the IVFSM for diagnosis of Dengue and Chikangunya.

In practical life, we find an amazing role of Matrices in vast areas of engineering and science. In spite of all it practicality, while the classical matrix theory is unable to solve the issues of many kinds of oddities. Mondal et al. [9] gave matrix illustration of an IVFS and SS to present the solutions of problems faced by matrix theory. In [7], Yong et al. applied it on decision making problems. Bohra et al. [10] further advanced the fuzzy soft matrix theory and its applicability in different fields of life. In [11] Chetia et al. applied intuitionistic fuzzy soft matrix for medical diagnosis and extended the algorithm operations. In [12] Rajarajeshwari and Dhanalakshmi introduced IVFSM and its kinds. In this paper we discuss the concept of IVFSM with examples and use it in decision making problem.

PRELIMINARIES

Definition 1

Let W and E represents the universe and set of parameters respectively and A be any subset of E then a pair (F, A) is called a SS over W and F is a mapping from A to P (W). It is not a set, but a parameterized family of subsets of the W [1].

Definition 2

Let W and E represents the universe and set of parameters respectively and A be any subset of E then a pair (F, A) is called FSS over W and F is a mapping from A to P (W), where P (W) is the collection of fuzzy subsets of W [4].

Definition 3

Let W and E represents the universe and set of parameters respectively and A be any subset of E then a pair (F, A) is called FSS in the fuzzy soft class (W, E) [15]. Then that FSS can be represented in matrix form such as $A_{mxn} = [a_{ij}]_{mxn}$ or $A = [a_{ij}]$ i=1, 2, …, m, j=1, 2, ….., n

Where

$$a_{ij} = \begin{cases} \mu_j(b_j) & if\ t_j \in A \\ 0 & \text{if } t_j \notin A \end{cases}$$

Definition 4

Let W and E represents the universe and set of parameters respectively and A be any subset of E then a pair (F, A) is called IVFSS over W where F is a mapping such that

$F: A \to I^W$

Where I^W represent the all interval valued fuzzy subsets (IVFSbS) of W [11].

Definition 5

Let W and E represents the universe and set of parameters respectively and A be any subset of E then a pair (F, A) is called IVFSS over W, where F is a mapping such that

$F: A \to I^W$, where I^W represent the all IVFSbS of W [13]. Then the IVFSS can be expressed in matrix form as

$A_{mxn} = [a_{ij}]_{mxn}$ or

$A = [a_{ij}]$ i = 1, 2, …, m, j = 1, 2, ..., n

Where

$$a_{ij} = \begin{cases} [\mu_{jL}(b_i), \mu_{jU}(b_i)] & \text{if } t_j \in A \\ [0,0] & \text{if } t_j \notin A \end{cases}$$

Where $[\mu_{jL}(b_i), \mu_{jU}(b_i)]$

Represent the membership of b_i in the IVFS F (e_j).

Example 1

Consider four provinces of Pakistan under consideration, namely the universe

W={ Q, P, K, S }, where Q, P, K and S are stands for Baluchistan, Punjab, Khyber Pakhtoonkhaw and Sindh respectively and E be a set of parameters $E=\{t_1, t_2, t_3, t_4\}$ where t_i stands for Education, health, local bodies and development program respectively. Consider

$A = \{t_1, t_2\} \subseteq E$ $F: A \to P(W)$

Consider an IVFSS (F, A) which describes the performance of provinces of Pakistan. Then IVFSS (F, A) is

(F, A) = {F (t_1) = {(Q, [0.6, 0.7]), (P, [0.7, 0.8]), (K, [0.8, 0.9]), (S, [0.4, 0.5])}, F (t_2) = {(Q, [0.4, 0.5]), (P, [0.6, 0.7]), (K, [0.8, 0.9]), (S, [0.3, 0.4])}}

This IVFSM can be represented as

$$(F, A) = \begin{matrix}Q\\P\\K\\S\end{matrix}\begin{bmatrix}[0.6, 0.7] & [0.4, 0.5] & [0.0, 0.0] & [0.0, 0.0]\\ [0.7, 0.8] & [0.6, 0.7] & [0.0, 0.0] & [0.0, 0.0]\\ [0.8, 0.9] & [0.8, 0.9] & [0.0, 0.0] & [0.0, 0.0]\\ [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0] & [0.0, 0.0]\end{bmatrix}$$

Definition 6

Let A and B are two interval valued fuzzy soft matrices (IVFSMs) then A is said to be interval valued fuzzy soft sub matrix of B if $\mu_{AL} \le \mu_{BL}$ and $\mu_{AU} \le \mu_{BU}$ for all i and j it is denoted by $A \subseteq B$ [13].

Example 2

Let A and B are any two subsets of E such that $A \subseteq B$

A= {t_1, t_2} and B = {t_1, t_2, t_3}

(F, A) = {F (t_1) = {(Q, [0.6, 0.7]), (P, [0.7, 0.8]), (K, [0.8, 0.9]), (S, [0.4, 0.5])},

F (t_2) = {(Q, [0.4, 0.5]), (P, [0.6, 0.7]), (K, [0.8, 0.9]), (S, [0.3, 0.4])}}

This IVFSS can be represented as

$$(F, A) = \begin{matrix}Q\\P\\K\\S\end{matrix}\begin{bmatrix}[0.6, 0.7] & [0.4, 0.5] & [0.0, 0.0] & [0.0, 0.0]\\ [0.7, 0.8] & [0.6, 0.7] & [0.0, 0.0] & [0.0, 0.0]\\ [0.8, 0.9] & [0.8, 0.9] & [0.0, 0.0] & [0.0, 0.0]\\ [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0] & [0.0, 0.0]\end{bmatrix}$$

(F, B) = {F (t_1) = {(Q, [0.6, 0.7]), (P, [0.7, 0.8]), (K, [0.8, 0.9]), (S, [0.4, 0.5])},

F (t_2) = {(Q, [0.4, 0.5]), (P, [0.6, 0.7]), (K, [0.8, 0.9]), (S, [0.3, 0.4])},

F (t_3) = {(Q, [0.3, 0.4]), (P, [0.2, 0.3]), (K, [0.5, 0.6]), (S, [0.2, 0.3])}}

$$(F, B) = \begin{matrix}Q\\P\\K\\S\end{matrix}\begin{bmatrix}[0.6, 0.7] & [0.4, 0.5] & [0.3, 0.4] & [0.0, 0.0]\\ [0.7, 0.8] & [0.6, 0.7] & [0.2, 0.3] & [0.0, 0.0]\\ [0.8, 0.9] & [0.8, 0.9] & [0.5, 0.6] & [0.0, 0.0]\\ [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] & [0.0, 0.0]\end{bmatrix}$$

Where (F, A) is soft sub matrix of (F, B)

Definition 7

Let A= [a_{ij}] be a IVFSM of order mxn, where [a_{ij}]=[$\mu_{jL}(b_i)$, $\mu_{jU}(b_i)$] then transpose of this IVFSM can be defined as [a_{ji}] , i = 1, 2, …….., m and j = 1, 2, ……., n of order n×m. It is denoted by A^T [13].

Example 3

Let A be IVFSM of order 2×2

$$A=\begin{bmatrix}[0.4, 0.6] & [0.3, 0.7]\\ [0.3, 0.5] & [0.5, 0.7]\end{bmatrix}$$ then its transpose can be written as

$$A^T=\begin{bmatrix}[0.4, 0.6] & [0.3, 0.5]\\ [0.3, 0.7] & [0.5, 0.7]\end{bmatrix}$$

Definition 8

Two IVFSMs A and B are said to conformable for addition if their order is same, it is defined as

A+B= [max (μ_{AL}, μ_{BL}), max (μ_{AU}, μ_{BU})] for all i and j [14].

Example4

Let A and B are two IVFSM then their addition is defined as

$$A=\begin{bmatrix}[0.4, 0.6] & [0.3, 0.7]\\ [0.3, 0.5] & [0.5, 0.7]\end{bmatrix}$$ and

$$B=\begin{bmatrix}[0.5, 0.6] & [0.6, 0.9]\\ [0.1, 0.7] & [0.5, 0.8]\end{bmatrix}$$

$$A+B = \begin{bmatrix}[0.5, 0.6] & [0.6, 0.9]\\ [0.3, 0.7] & [0.5, 0.8]\end{bmatrix}$$

Definition 9

Two IVFSMs A and B are said to conformable for subtraction if their order is same, it is defined as

A-B= [min (μ_{AL}, μ_{BL}), min (μ_{AU}, μ_{BU})] for all i and j [13].

Example 5

Let A and B are two IVFSMs then their subtraction is defined as

$$A=\begin{bmatrix}[0.4, 0.6] & [0.3, 0.7]\\ [0.3, 0.5] & [0.5, 0.7]\end{bmatrix}$$ and

$$B=\begin{bmatrix}[0.5, 0.6] & [0.6, 0.9]\\ [0.1, 0.7] & [0.5, 0.8]\end{bmatrix}$$

$$A-B = \begin{bmatrix}[0.4, 0.6] & [0.3, 0.7]\\ [0.1, 0.5] & [0.5, 0.7]\end{bmatrix}$$

Definition 10

Let A= [a_{ij}] and B= [b_{ij}] are two IVFSMs of order m×n and n×p respectively then their product defined as A*B = [c_{ik}]$_{m\times p}$= [max min (μ_{ALj}, μ_{BLj}), max min (μ_{AUj}, μ_{BUj})] [14].

Example 6

Let A and B are two IVFSMs then their product is defined as

$$A=\begin{bmatrix}[0.4, 0.6] & [0.3, 0.7]\\ [0.3, 0.5] & [0.5, 0.7]\end{bmatrix}$$ and

$$B=\begin{bmatrix}[0.5, 0.6] & [0.6, 0.9]\\ [0.1, 0.7] & [0.5, 0.8]\end{bmatrix}$$

$$A*B = \begin{bmatrix}[0.4, 0.6] & [0.3, 0.7]\\ [0.3, 0.5] & [0.5, 0.7]\end{bmatrix} * \begin{bmatrix}[0.5, 0.6] & [0.6, 0.9]\\ [0.1, 0.7] & [0.5, 0.8]\end{bmatrix}$$

$$A*B = \begin{bmatrix}[0.4, 0.6] + [0.1, 0.7] & [0.4, 0.6] + [0.3, 0.7]\\ [0.3, 0.5] + [0.1, 0.7] & [0.3, 0.5] + [0.5, 0.7]\end{bmatrix}$$

$$A*B = \begin{bmatrix}[0.4, 0.7] & [0.4, 0.7]\\ [0.3, 0.7] & [0.5, 0.7]\end{bmatrix}$$

Definition 11

Let A= [a_{ij}] be an IVFSM of order m×n, where a_{ij} = [μ_{Jl} (b_i), μ_{jU} (b_i)]

Then complement of A is defined as A^C= [bij] where b_{ij} = [1-$\mu_{jU}(b_i)$, 1-$\mu_{jL}(b_i)$] for all i, j [14].

Example 7

Let A be an IVFSM.

$$A=\begin{bmatrix}[0.4, 0.6] & [0.3, 0.7]\\ [0.3, 0.5] & [0.5, 0.7]\end{bmatrix}$$

Then its complement is defined as

$$A^C = \begin{bmatrix}[0.4, 0.6] & [0.3, 0.7]\\ [0.5, 0.7] & [0.3, 0.5]\end{bmatrix}$$

And see other related definition in [13].

Generalized IVFSMs in decision making problem

Two decision makers (Pildat and Public survey) want to take a decision that which province is best from all provinces of Pakistan according to these parameters such as education, health, local bodies and devolvement programs i.e., parameters (E). Each decision maker has freedom to take decision and evaluation of parameters related to the chosen object or may option the same set of parameters. Here it is assumed that the evaluation of parameters by the decision makers must be generalized fuzzy and may be presented in linguistic form or generalized FSS format, alternatively, in the form of generalized fuzzy soft matrices. Now the aim of the decision makers is to find which province of Pakistan has best governance in 2015.

APPLICATION OF IVFSMS IN DECISION MAKING

Algorithm

1. Select the appropriate subsets of the set of parameters.
2. Construct the IVFSMs with respect to the parameters of the decision maker.
3. Compute the product of IVFSMs as per rule of multiplication of IVFSM.
4. Compute the row wise sum of these products of IVFSM.
5. Now we find which row has the greatest interval.

Let W be the universal set which contains four provinces of Pakistan such that

W = {Q, P, K, S} where Q, P, K and S represent Baluchistan, Punjab, Khyber Pakhtonkhaw and Sindh respectively. Let E be the set of parameter such that E= $\{t_1, t_2, t_3, t_4\}$ where t_i stand for

Education, health, local bodies and development program, respectively. Where we determine which province has good governance according to above parameters in Pildat and public survey 2015.

A (Pildat survey)

$$= \begin{matrix} Q \\ P \\ K \\ S \end{matrix} \begin{bmatrix} [0.4, 0.5] & [0.3, 0.4] & [0.5, 0.6] & [0.2, 0.3] \\ [0.7, 0.8] & [0.6, 0.6] & [0.2, 0.3] & [0.5, 0.6] \\ [0.8, 0.8] & [0.7, 0.8] & [0.5, 0.6] & [0.4, 0.5] \\ [0.3, 0.4] & [0.3, 0.4] & [0.2, 0.3] & [0.2, 0.3] \end{bmatrix}$$

$$\text{B (Public survey)} = \begin{matrix} Q \\ P \\ K \\ S \end{matrix} \begin{bmatrix} [0.6, 0.7] & [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] \\ [0.7, 0.8] & [0.6, 0.7] & [0.2, 0.3] & [0.3, 0.4] \\ [0.8, 0.9] & [0.8, 0.9] & [0.5, 0.6] & [0.4, 0.5] \\ [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] & [0.2, 0.3] \end{bmatrix}$$

$$A*B= \begin{bmatrix} [0.4, 0.5] & [0.3, 0.4] & [0.5, 0.6] & [0.2, 0.3] \\ [0.7, 0.8] & [0.6, 0.6] & [0.2, 0.3] & [0.5, 0.6] \\ [0.8, 0.8] & [0.7, 0.8] & [0.5, 0.6] & [0.4, 0.5] \\ [0.3, 0.4] & [0.3, 0.4] & [0.2, 0.3] & [0.2, 0.3] \end{bmatrix} * \begin{bmatrix} [0.6, 0.7] & [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] \\ [0.7, 0.8] & [0.6, 0.7] & [0.2, 0.3] & [0.3, 0.4] \\ [0.8, 0.9] & [0.8, 0.9] & [0.5, 0.6] & [0.4, 0.5] \\ [0.4, 0.5] & [0.3, 0.4] & [0.2, 0.3] & [0.2, 0.3] \end{bmatrix}$$

$$A*B= \begin{matrix} Q \\ P \\ K \\ S \end{matrix} \begin{bmatrix} [0.5, 0.6] & [0.5, 0.6] & [0.4, 0.5] & [0.4, 0.5] \\ [0.6, 0.7] & [0.6, 0.6] & [0.3, 0.4] & [0.3, 0.4] \\ [0.7, 0.8] & [0.6, 0.7] & [0.5, 0.6] & [0.4, 0.5] \\ [0.3, 0.4] & [0.3, 0.4] & [0.3, 0.4] & [0.3, 0.4] \end{bmatrix}$$

$$\mathbf{A}*B= \begin{matrix} Q \\ P \\ K \\ S \end{matrix} \begin{bmatrix} [0.5, 0.6] \\ [0.6, 0.7] \\ [0.7, 0.8] \\ [0.3, 0.4] \end{bmatrix}$$

It is clear from above matrix that interval in third row have maximum value. So Khyber Pakhtonkhaw has good governance in 2015 according to selected parameters.

CONCLUSION

From the above discussion, we discovered the concept of IVFSM, we defined different types of IVFSMs with their illustrations, we used the IVFSMs in decision making too. In future, these special operators will be used for facilitation and smooth application of decision making in different fields of life. We shall also be able to compare the proven results with other methods of decision making.

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