# POISSON NOISE REMOVAL USING WAVELET TRANSFORMS

**Shahzad Khursheed[1], Amir A Khaliq[2], Suheel A Malik[3],Jawad A Shah[4], Sheroz Khan[5]**
[1,2,3,4] Department of Electronic Engineering, Faculty of Engineering &Technology, International Islamic University Islamabad, Pakistan
[5]Visting Faculty, from Dept. of ECE, International Islamic University Malaysia
[1]shahzad.msee280@iiu.edu.pk, [2]m.amir@iiu.edu.pk, [3]suheel.abdullah@iiu.edu.pk, [4]Jawad.shah@iiu.edu.pk,
[5]cnar32.sheroz@gmail.com

***ABSTRACT:*** *Medical images play a vital role in diagnosing different diseases. During acquisition process image acquired by diverse imaging modalities may get worse by different types of noises. Poisson noise is one of the complicated noises, which is very difficult from de-noising point of view. For de-noising medical images different techniques are used, wavelet transform is one of them. In this work discrete wavelet transforms are used for Poisson noise removal using modified thresholding function on general images as well as X-Rays, PET and SPECT images. Results of wavelet-modified filters are compared to other conventional filters in terms of correlation, Peak signal-to-noise ratio (PSNR), mean structural similarity index measure (MSSIM).*

**Keywords:** Discrete Wavelet Transforms, De-Noising By DWT, Thres

## INTRODUCTION

Medical imaging has been getting a continuously increasing importance as part of medical diagnosis over the last few decades. Several medical imaging modalities are developed to acquire the image for examination purpose of the parts of an entire body without surgical procedures being carried out. During such process image acquired may get infested by several types of noise-ridden sources, such as ultrasound image by speckle noise, Magnetic Resonance Imaging (MRI) by Rician noise, and X-rays, Positron Emission Tomography (PET)[1], Single photon Emission Computed Tomography (SPECT) by Poisson noises[2, 3].

One of the most pre-processing steps in the field of medical imaging is noise removal. Most of de-noising algorithms are developed for removal of Additive White Gaussian Noise (AWGN). Due to additive nature of AWGN, it can be easily removed from images and also literature is abundant for AWGN de-noising. On the other hand the removal of non-Gaussian noise such as Poisson noise, Rician noise, is much difficult due to its multiplicative and signal dependent nature[4]. Therefore, literature on de-noising of medical images corrupted by Poisson noise and Rician noise is very limited.

For de-noising medical images from different modalities of noise, a number of techniques have been used in literature. In this research work, our focus is to de-noise medical images corrupted by Poisson noise only, using wavelet transforms with modified thresholding. In this work Discrete wavelet transforms (DWT) is applied on general as well as different medical images corrupted by Poisson noise up to certain levels. After getting sub-bands, modified threshold is applied on each sub-band. Restoration of de-noised image is performed by applying inverse wavelet transforms. The main contribution of this work is the modified thresholding techniques which out performs from the conventional filters. The details of discrete wavelet transforms (DWT) and modified threshold is discussed in coming section.

### Signal Model

Equation (1) gives the mathematical model for degradation and restoration purpose [5]

$$g(u,v) = s(u,v) * h(u,v) + n(\mathrm{u,v}) \quad (1)$$

Where $g(u,v)$define noisy image, $h(u,v)$ is blurring kernel, $s(u,v)$ original signal which is required to be recover and $n(\mathrm{u,v})$ is noise. For de-noising purpose, blurring kernel is neglected and equation (2) represents the degradation model without blurring kernel [5]

$$g(u,v) = s(u,v) + n(\mathrm{u,v}) \quad (2)$$

Equation (3) gives the mathematical model for multiplicative noise[5]

$$g(u,v) = s(u,v).\, n(u,v) \quad (3)$$

### Poisson Noise

Poisson noise is an electronic noise that happens when limited quantity of particles that contain energy, for example, electrons or photons in an electronic circuit or in optical devices respectively, is little enough to produce some detectable statistical variations in estimation. Poisson noise is also known as short or photon noise.

Poisson Noise is a multiplicative noise having an uncertain nature, which is associated with the estimation of light. The magnitude of Poisson noise is signal dependent and is considered as a high source of image noise but not in a case of low light conditions.

Let $m = \{m_{i,j} : i,j = 1, \dots \dots \dots N\}$
and $n = \{n_{i,j} : i,j = 1, \dots \dots \dots N\}$ denote the original and noisy images respectively, whereas the noisy image values are contaminated by Poisson noise. For a given true image $m$, the likelihood for observing $n$ is represented in equation (4)

$$p(n/m) = \prod_{i,j=1}^{N} \frac{e^{-m_i} m_{i,j}^{n_{i,j}}}{n_{i,j}!} \quad (4)$$

In this work we have used the discrete wavelet transform for noise removal with a modified thresholding function. The work is closely related to paper [5].

### Discrete Wavelet Transform Filter for Poisson Noise:

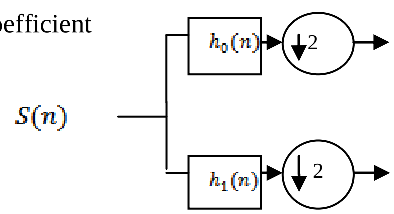
Wavelet transform is Bayesian approach for estimation of Poisson intensity, relying on un-normalized Haar wavelet Transform [6, 7]. Mallat proposed multi-resolution technique for signal representation which follow the idea of wavelet decomposition[8, 9], the discrete wavelet transform (DWT) got importance as an essential tool in both signal and image processing. The DWT, is linear transformation, which is based on sub-Band Coding and function or operates on data vectors having length of integer ($n$) power of two ($2^n$), and translate it into different numerical vectors having same length. DWT divides data into different frequency components and, analyze these frequencies with different scales. DWT can be implemented by means of cascade of high pass and low pass filters, output of each filter is then sub-sampled by factor of 2. The wavelets provide thus a set of basis, which can be used to represent data set, the high pass filter and low pass filter represent data set in the form of differences and average values, called detailed coefficient and approximate coefficient respectively.

If $S(n)$ be the signal, $h_0(n)$ be the impulse response of low pass filter and $h_1(n)$ be the impulse response of high pass filter, then the DWT of $S(n)$ can be computed by passing it through low pass filter with output, $y_0(n)$, given in form of convolution of $h_0(n)$ and $S(n)$. Also, by passing it the same signal through high pass filter having impulse response of $h_1(n)$, such that its output, $y_1(n)$, becomes the convolution of $h_1(n)$ and $S(n)$. Proceeding like this, the DWT can be mathematically calculated by equation (6) and (7)[10]. Figure 1 shows the implementation of 1D DWT.

$$y_0(n) = S(n) * h_0(n) = \sum_{k=-\infty}^{\infty} S(k) * h_0(n-k) \quad (6)$$

$$y_1(n) = S(n) * h_1(n) = \sum_{k=-\infty}^{\infty} S(k) * h_1(n-k) \quad (7)$$

Approximation Coefficient




Detail coefficients

**Figure 1:** One-Dimensional (1D) DWT Analysis Filter

According to Nyquist's rule half of samples of output of both filters must be discarded. So the outputs of filters are sub-sampled by 2. Therefore, resultant output is calculated by equations (8) and (9)

$$y_0(n) = \sum_{k=-\infty}^{\infty} S(k) * h_0(2n-k) \quad (8)$$

$$y_1(n) = \sum_{k=-\infty}^{\infty} S(k) * h_1(2n+1-k) \quad (9)$$

As images are two dimensional, so 2D DWT is required to apply on an image. The implementation of 2D DWT can be achieved by applying One-Dimensional (1D) DWT in both columns and rows direction shown in Figure 2. The wavelet transform decomposes an image into four sub-bands LL, LH, HL, HH, here L define the result of low pass filter and H define the result of high pass filter. The LL band is known as Approximation coefficient, whereas LH, HL and HH bands are known as details coefficients. LH, HL and HH bands hold horizontal, vertical and diagonal details respectively[10, 11]. For de-noising purpose thresholding is applied on coefficients. There are some conventional thresholding techniques, which are mostly used for de-noising purpose such as hard threshold and soft threshold[12]. In the case of hard threshold follows the rule of "keep or kill", means that if absolute of coefficient is less than threshold value then it is replaced with zero and if absolute of coefficient is greater than threshold value then it retained without any change. Mathematically hard threshold is calculated by equation (10).

$$y_{j,k} = \begin{cases} w_{j,k} & |w_{j,k}| \geq T \\ 0 & |w_{j,k}| < T \end{cases} \quad (10)$$

Whereas in the case of soft thresholding, threshold value is subtracted from coefficient if absolute of coefficient is greater than or equal to threshold value and coefficient is replaced with resultant value. If absolute of coefficient is less than threshold value, then coefficient is replaced with zero. If coefficient is less than or equal to negative value of threshold then coefficient is replaced with value of sum of coefficient and threshold. Mathematically soft threshold is calculated by equation (11).

$$y_{j,k} = \begin{cases} w_{j,k} - T & |w_{j,k}| \geq T \\ 0 & |w_{j,k}| < T \\ w_{j,k} + T & w_{j,k} \leq -T \end{cases} \quad (11)$$

The condition of equation (11) can also be implemented by equation (12)

$$y_{j,k} = \begin{cases} sgn(w_{j,k})(|w_{j,k}| - T) & |w_{j,k}| \geq T \\ 0 & |w_{j,k}| < T \end{cases} \quad (12)$$

However, modified threshold implemented in this paper work relay on equation (13)

$$y_{j,k} = \begin{cases} w_{j,k}\left(\frac{|w_{j,k}|-T}{(|w_{j,k}|-T)+T}\right) & |w_{j,k}| > T \\ 0 & |w_{j,k}| \leq T \end{cases} \quad (13)$$

In modified threshold, if absolute of coefficient is greater than the threshold value then coefficient is replaced by value calculated by above given formula, and if absolute of coefficient is less than or equal to threshold value then it is replaced with zero.

Figure 2 shows the complete de-noising process of an image corrupted by non-Gaussian noise. First of all corrupted image is decomposed into coefficients by applying 1D DWT along both columns and rows direction. After decomposition modified threshold is applied on all coefficients. For the reconstruction of the image, the inverse discrete wavelet transform in applied on the resultant coefficients of modified threshold.

**Results and Discussion**

In this work de-noising is performed on LENA, PET, SPECT and X-Rays images corrupted by Poisson noise by using discrete wavelet transform and complex discrete wavelet transform algorithms. The results of modified thresholding DWT (Discrete Wavelet Transforms) and modified thresholding Complex Dual Tree DWT (CDTDWT) are compared with the results obtained by using Median filter and Wiener filter.

Figure 3 (a) shows original images of LENA, PET, SPECT and X-Rays in row 1and noisy images of LENA, PET, SPECT and X-Rays in row 2, whereas images in row 3 of Figure 3(a) are de-noised by DWT. Noisy images de-noised by CDT-DWT in row 1, Median filter in row 2, and Wiener filter in row 3, are as shown in Figure 3(b).

From visual quality of images it can be seen that performance of DWT and CDT-DWT is better than median and Wiener filters. It can be seen further that both DWT and CDT-DWT are smoothing noise excellently while preserving essential structure like edges and also provide homogeneity with original image. Whereas by visual quality of image de-noised by Median filter (as shown in Figure-3b), it can be seen that Median filter have low tendency for completely removing Poisson noise. Moreover, de-noising by Wiener filter doesn't preserve the image structure and blur it and noise from edges also doesn't remove.

In case of LENA image de-noising results show that performance of Discrete Wavelet Transform (DWT) is much better than other filter as PSNR, CORR and MSSIM values of DWT is much greater than other and the performance of Complex Dual Tree DWT (CDTDWT) is also better than Median and Wiener filters.

In case of PET image de-noising results show that performance of Discrete Wavelet Transform (DWT) is better than Complex Dual Tree DWT (CDTDWT), Median Filter and Wiener Filter, in term of PSNR and Correlation, whereas MSSIM values of DWT and CDTDWT are very close but better than Median and Wiener filter.

In case of SPECT image de-noising results shows that performance of DWT is much better than CDTDWT, Median Filter and Wiener Filter in term of PSNR, Correlation and MSSIM. However, the performance of CDTDWT is closely same to Median filter for low level of Poisson noise but better for high value of noise.

In case of X-Rays image de-noising results shows that Discrete Wavelet Transform (DWT) is performing in term of PSNR from Complex Dual Tree DWT (CDTDWT), Median Filter and Wiener Filter. Correlation and MSSIM values of DWT and CDTDWT are very close but better than Median and Wiener filter.

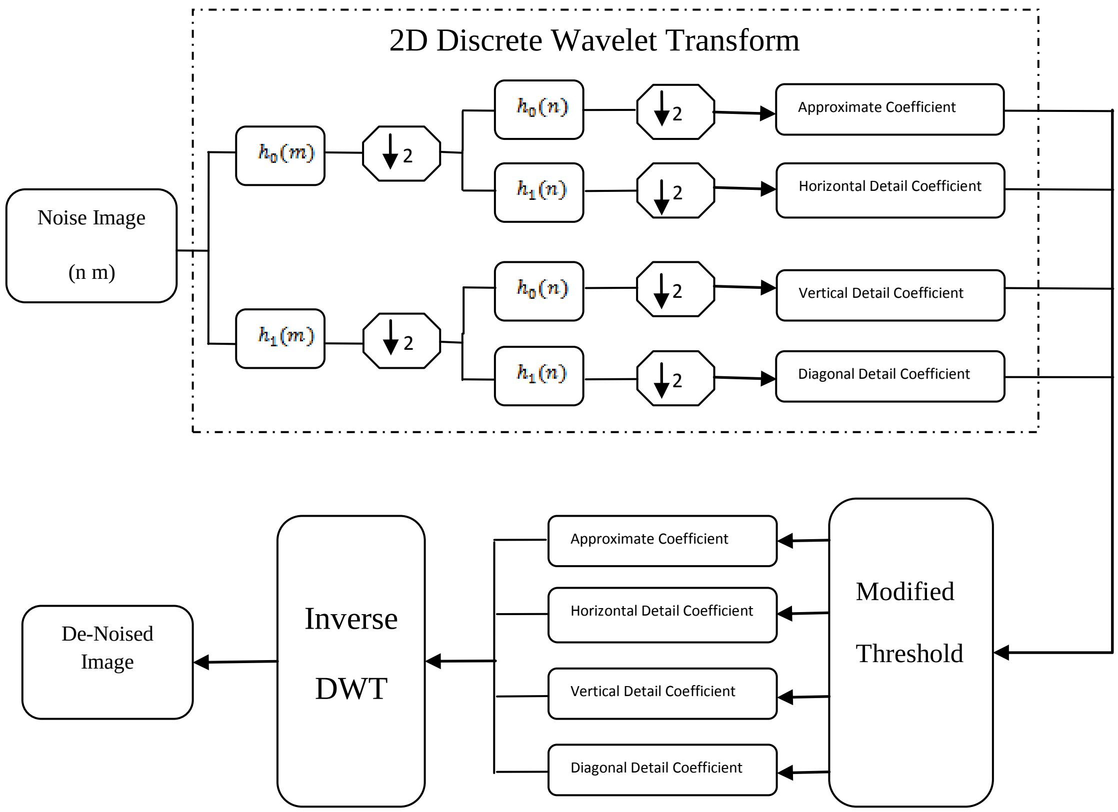



**Figure 2: Process for Image De-Noising By Two Dimensional (2D) DWT**

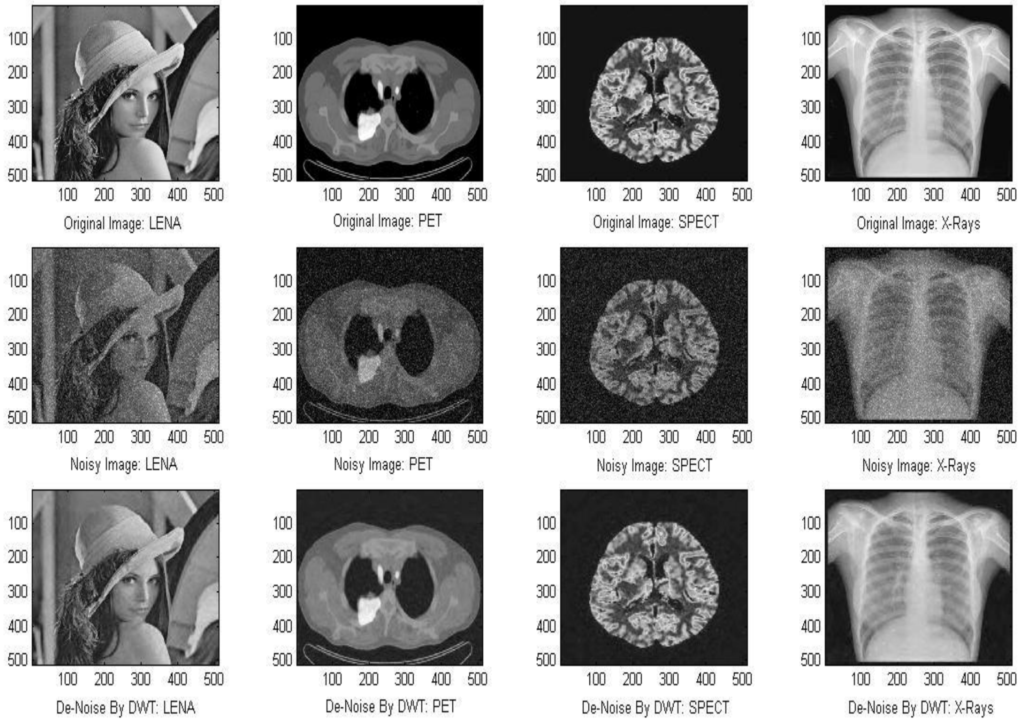



**Figure 3(a):** Original Image, Noisy Image, De-noise by DWT

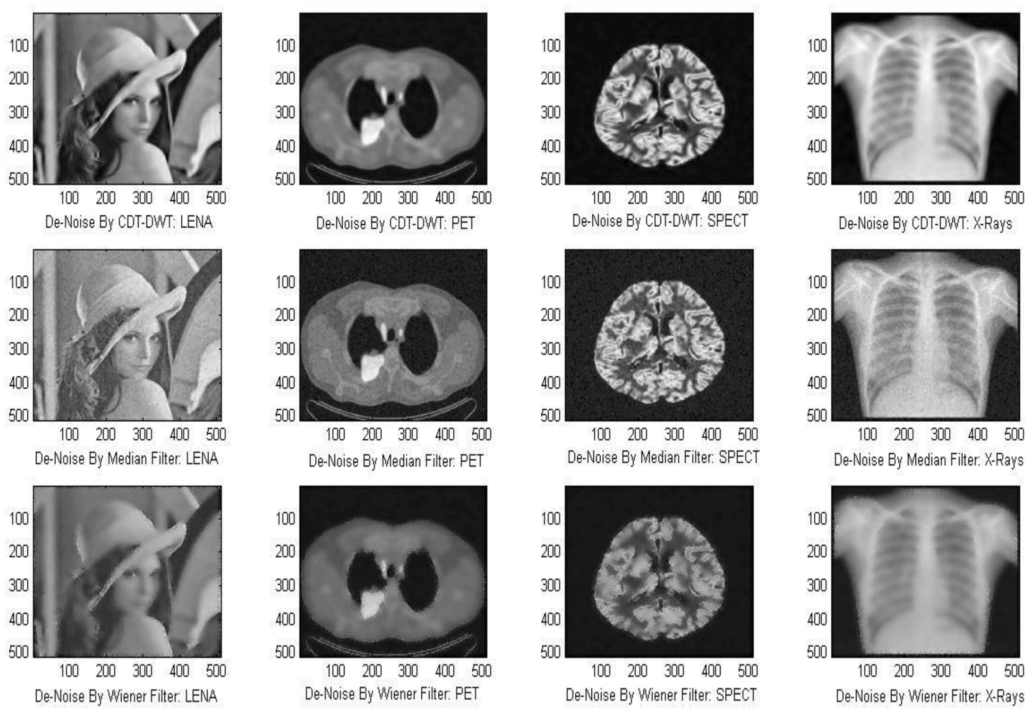



**Figure 3(b): Images De-Noised By CDT-DWT, Median Filter, Wiener Filter**

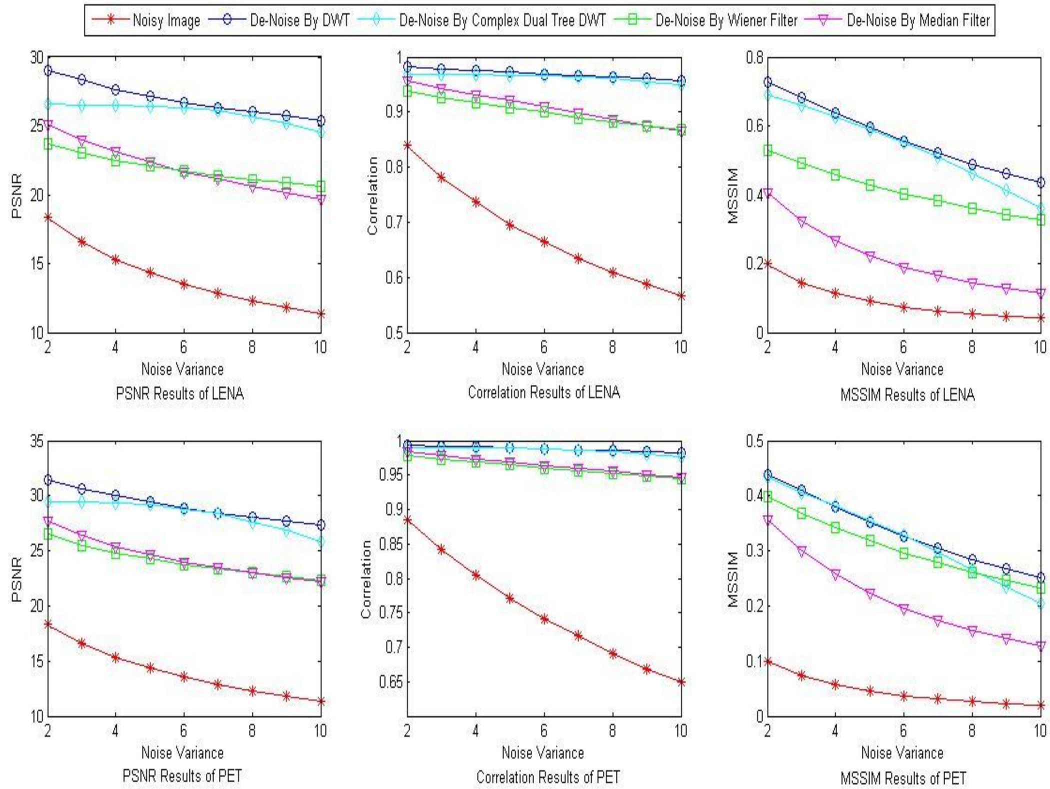



**Figure 4(a): Comparative Performance of De-Noising Methods for LENA and PET**

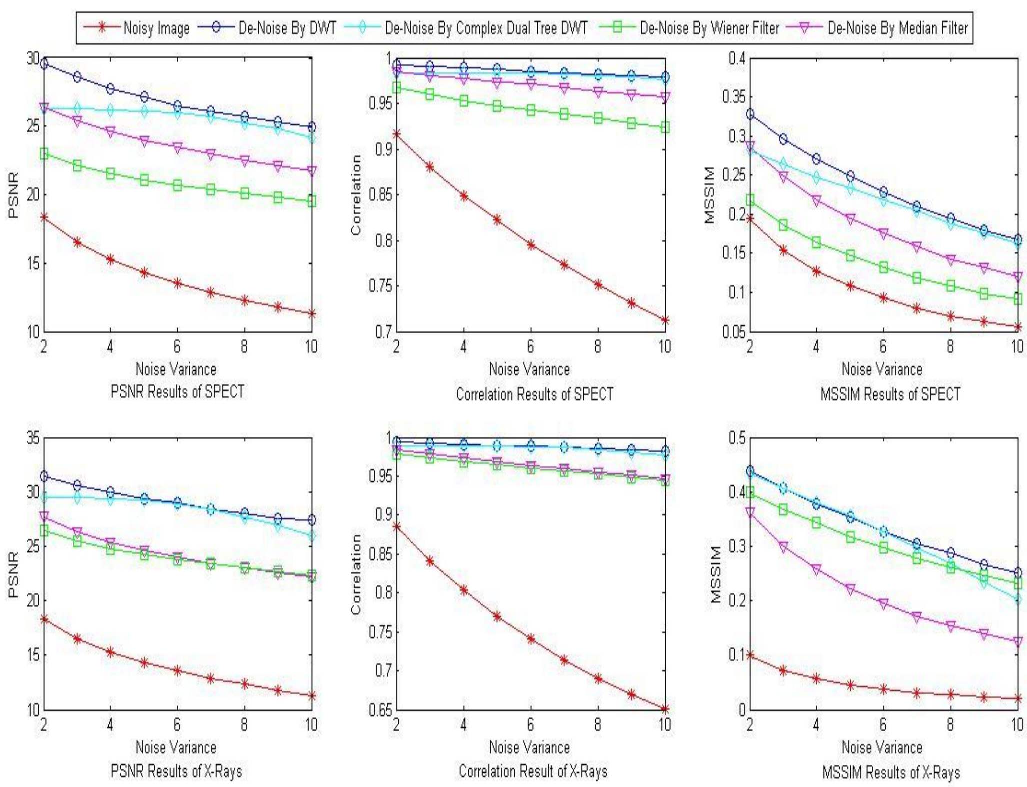



**Figure 4(b): Comparative Performance of De-Noising Methods for PECT and X-Rays**

**Table1: PSNR, CORR, MSSIM Results of DWT, CDTWT, Median and Wiener filter of LENA**

| | LENA IMAGE | | | | |
|---|---|---|---|---|---|
| | Noisy image | DWT | CDTDWT | Median Filter | Wiener Filter |
| PSNR | 18.31795 | 29.0877 | 26.555 | 25.0799 | 23.6567 |
| CORR | 0.8388 | 0.9823 | 0.9682 | 0.9558 | 0.9371 |
| MSSIM | 0.1983 | 0.7274 | 0.6981 | 0.5456 | 0.6107 |

**Table 2: PSNR, CORR, MSSIM Results of DWT, CDTWT, Median and Wiener filter of PET**

| | PET Image | | | | |
|---|---|---|---|---|---|
| | Noisy image | DWT | CDTDWT | Median Filter | Wiener Filter |
| PSNR | 18.3263 | 31.3705 | 29.4752 | 27.6364 | 26.4927 |
| CORR | 0.8864 | 0.9932 | 0.9896 | 0.9843 | 0.9736 |
| MSSIM | 0.1005 | 0.4369 | 0.4368 | 0.3572 | 0.3987 |

**Table 3:** PSNR, CORR, MSSIM Results of DWT, CDTWT, Median and Wiener filter of SPECT

| | SPECT Image | | | | |
|---|---|---|---|---|---|
| | Noisy image | DWT | CDTDWT | Median Filter | Wiener Filter |
| PSNR | 18.3065 | 29.5627 | 26.2767 | 26.4057 | 23.0330 |
| CORR | 0.9155 | 0.9928 | 0.9848 | 0.9850 | 0.9675 |
| MSSIM | 0.1942 | 0.3275 | 0.2764 | 0.2596 | 0.2176 |

**Table 4: PSNR, CORR, MSSIM Results of DWT, CDTWT, Median and Wiener filter of X-Rays**

| | X-Rays Image | | | | |
|---|---|---|---|---|---|
| | Noisy image | DWT | CDTDWT | Median Filter | Wiener Filter |
| PSNR | 18.2984 | 31.4680 | 29.4899 | 27.7290 | 26.4732 |
| CORR | 0.9273 | 0.9934 | 0.9938 | 0.9844 | 0.9789 |
| MSSIM | 0.1310 | 0.4317 | 0.4366 | 0.3622 | 0.3981 |

## CONCLUSION:

From the de-nosing results of all filters, it can be concluded that overall performance of DWT and CDTDWT is admirable in LENA, PET, SPECT and X-Rays. However in PET Image the performance of DWT and CDTDWT is very close in term of Correlation and MSSIM. In SPECT, CDTDWT performance is not better for low value of noise but overall performance of CDTDWT is much better. Also DWT is performing very well in SPECT.

## REFERENCES


[1]. Ollinger J. M. and Fessler J. A., Positron Emission Tomography,*IEEE Signal Process. Mag.*, **14(1)**, 43–55, (1997)

[2]. Unser M. and Aldroubi A., A Review of Wavelets in Biomedical Applications, *Proc. IEEE*,**84(4)**, 626-638, (1996)

[3]. RangarajRangayyan M., Biomedical Signal Analysis: A Case Study Approach, *IEEE Process. Mag.*,**19(4)**, (2002)

[4]. SanchesJo˜ao M., Nascimento Jacinto C. and Marques Jorge S., An Unified Framework for Bayesian De-noising for Several Medical and Biological Imaging Modalities, *IEEE Int. Conf.*,6267-6270, (2007)

[5]. Naga Prudhvi Raj V. and Venkateswarlu T., De-noising of Poisson and Rician Noise from Medical Images using Variance Stabilization and Multi-scale Transforms, *Int. J.Comput.Appl.*,57**(21)**, 0975-8887 (2012)

[6]. Willett R. M. and Nowak R. D., Fast Multi-Resolution Photon-Limited Image Reconstruction, Biomedical Imaging: Nano to Macro, *IEEE Int.Symp.*,**2**, 1192–1195 (2004)

[7]. Rodrigues I., Sanches J. and Bioucas-Dias J., De-noising of Medical Images Corrupted by Poisson Noise, *IEEE Int. Conf.*, 1756 – 1759(2008)

[8]. Mallat S., A Theory for Multi-Resolution Signal Decomposition: The Wavelet Representation, *IEEE Trans.*, **11(7**), 674-693 (1989)

[9]. Daubechies I., the Wavelet Transform Time-Frequency Localization and Signal Analysis, *IEEE Trans.*, **36(5)**, 961-1005 (1990)

[10]. StephaneMallat., A Wavelet Tour of Signal Processing, *Elsevier*,(2006)

[11]. Velmurugan A.K. and Kannan R.J., Wavelet analysis for medical image denoising based on thresholding techniques, *IEEE Int. Conf.*, 213 – 215(2013)

[12]. Donoho D. L., De-Noising by Soft Thresholding, *IEEE Trans. Inform. Theory*, **41(3)**, 613-627, (1995)