GENERALIZED EXPONENTIAL TYPE REGRESSION ESTIMATOR BY USING TWO AUXILIARY INFORMATION

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ABSTRACT: *In this paper, generalized exponential type regression estimators under simple random sampling without replacement has been proposed for estimating the finite population mean of study variable by using two auxiliary information in single phase sampling. The mean square error of proposed estimator is obtained up to first order approximation and bias expressions have been derived up to second order approximation. Under the theoretical condition proposed estimators are better than some existing estimators. An empirical study has been derived to demonstrate the efficiency of proposed estimators.*

Key words*:* Regression type estimator, auxiliary variable exponential estimators, Single-phase sampling mean square error, bias.

1. INTRODUCTION

In single phase sampling one of the most popular methods of estimation is regression method of estimation. Regression method of estimation incorporates auxiliary information and provides estimates that are efficient and precise. Auxiliary information assists in increasing the precision of the estimate. Grant [5] was the first who estimated the population of England based on auxiliary information. The work of Neyman [10] may be referred as an initial work where auxiliary information has been discussed in detail. Cochran [2] derived the classical ratio type estimator for the population mean and Cochran [3] prescribed auxiliary information in regression estimators. Robson [11] and Murthy [9] prescribed; if the correlation is negative the product method of estimation is quite effective. Mohanty's [7] utilized two auxiliary variables by combining the regression and ratio method. Bahl and Tuteja [2] were the first, who used exponential type estimators for estimating the population mean. Samiuddin and Hanif [12] proposed ratio and regression estimation procedure by using two auxiliary variables and they have provided modification of Mohanty [7] estimator. Hanif et al [6] investigated an estimator which was the modification of Singh and Espejo [14] estimator. The development is continued in the form of exponential estimators for different situations by many authors such as Singh and Vishwakarma [15], Noor ul amin and Hanif [8] and Sanaullah et al. [13].

2. NOTATIONS AND VARIOUS EXISTING ESTIMATORS

$$\left.\begin{aligned} &e_{\bar{y}}=\frac{\bar{y}-\bar{Y}}{\bar{Y}},\quad e_{\bar{x}}=\frac{\bar{x}-\bar{X}}{\bar{X}},\ e_{\bar{z}}=\frac{\bar{z}-\bar{Z}}{\bar{Z}}\\ &E(e_{\bar{y}})=E(e_{\bar{x}})=E(e_{\bar{z}})=0,\ E(e_{\bar{y}}^2)=\theta C_y^2,\ E(e_{\bar{x}}^2)=\theta C_x^2,\\ &E(e_{\bar{x}}e_{\bar{y}})=\theta\,\rho_{xy}C_xC_y,\ E(e_{\bar{y}}e_{\bar{z}})=\theta\,\rho_{yz}C_zC_y,\\ &\theta=\frac{1}{n}-\frac{1}{N},\ C_y=\frac{S_y}{\bar{Y}},\quad \rho_{xy}=\frac{S_{xy}}{S_xS_y},\ H_{ij}=\rho_{ij}\frac{C_i}{C_j}. \end{aligned}\right\}\quad (2.1)$$

(i) Cochran [2] and Robson [11] derived the classical ratio and product estimators, respectively, for estimating the population mean as

$$t_1=\bar{y}\left[\frac{\bar{X}}{\bar{x}}\right]. \quad (2.2)$$

$$t_2=\bar{y}\left[\frac{\bar{z}}{\bar{Z}}\right]. \quad (2.3)$$

The mean square equations (MSE) of the estimators of t_1 and t_2 are

$$MSE(t_1)\approx\bar{Y}^2\theta\left[C_y^2+C_x^2\left(1-2H_{yx}\right)\right] \quad (2.4)$$

$$MSE(t_2)\approx\bar{Y}^2\theta\left[C_y^2+C_z^2\left(1+2H_{yz}\right)\right] \quad (2.5)$$

(ii) Cochran [3] proposed the classical regression estimator given by

$$t_3=\bar{y}_r+b_{yx}\left(\bar{X}-\bar{x}\right).$$

(2.6) The mean square error of t_3 is given by

$$MSE(t_3)=\theta\bar{Y}^2C_y^2\left(1-\rho_{xy}^2\right). \quad (2.7)$$

(iii) Bahl and Tuteja [2] suggested the following exponential ratio and product type Estimators

$$t_4=\bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right). \quad (2.8)$$

$$t_5=\bar{y}\exp\left(\frac{\bar{z}-\bar{Z}}{\bar{z}+\bar{Z}}\right). \quad (2.9)$$

respectively, the mean square error of the estimators t_4 and t_5 are given by

$$MSE(t_4)\approx\theta\bar{Y}^2\left[C_y^2+\frac{C_x^2}{4}\left(1+4H_{yx}\right)\right] \quad (2.10)$$

$$MSE(t_5)\approx\theta\bar{Y}^2\left[C_y^2+\frac{C_z^2}{4}\left(1-4H_{yz}\right)\right] \quad (2.11)$$

respectively, the bias expressions of the estimators t_4 and t_5 are given by

$$Bias(t_4) \approx \frac{\bar{Y}\theta C_x^2}{8}\left[3-4H_{yx}\right]$$

(2.12) $$Bias(t_5) \approx \frac{\bar{Y}\theta C_x^2}{8}\left[4H_{yx}-1\right]. \quad (2.13)$$

(iv) Mohanty [7] proposed the Regression-Cum-Ratio Estimator for estimating the finite population mean

$$t_6 = \left[\bar{y} + b_{yx}\left(\bar{X}-\bar{x}\right)\frac{\bar{Z}}{\bar{z}}\right]. \quad (2.14)$$

The mean square error and bias expressions for t_6 are given by

$$MSE(t_6) = \theta\bar{Y}^2\left[C_y^2\left(1-\rho_{xy}^2\right)+C_z^2-2\rho_{yz}C_yC_z+2\rho_{xy}\rho_{xz}C_yC_z\right] \quad (2.15)$$

$$Bias(t_6) = \theta\bar{Y}C_z^2 - \theta\bar{Y}\rho_{yz}C_yC_z + b_{yx}\bar{X}\rho_{xz}C_xC_z. \quad (2.16)$$

respectively.

(v) The modification of Mohanty [7] have given by Samiuddin and Hanif [12] as

$$t_7 = \left[\bar{y} + k_1\left(\bar{X}-\bar{x}\right)\frac{\bar{Z}}{\bar{z}}\right], \quad (2.17)$$

The mean square error and bias expression of t_7 is given by

$$MSE(t_7) \approx \theta\bar{Y}^2\left[C_y^2\left(1-\rho_{xy}^2\right)+C_z^2\left(1-\rho_{xz}^2\right)-2C_yC_x\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right)\right]$$

(2.18) $$Bias(t_7) = \theta\bar{Y}C_z^2 - \theta\bar{Y}\rho_{yz}C_yC_z + b_{yx}\bar{X}\rho_{xz}C_xC_z$$

(vi) Samiuddin and Hanif [12] were using the idea of Chand [4] and suggested the estimator given by

$$t_8 = \bar{y}\frac{\bar{X}}{\bar{x}}\frac{\bar{Z}}{\bar{z}}. \quad (2.19)$$

The mean square error and bias expression of t_8 is given by

$$MSE(t_8) \approx \theta\bar{Y}^2\left(C_x^2+C_y^2+C_z^2-2C_xC_y\rho_{xy}-2C_yC_z\rho_{yz}+2C_xC_z\rho_{xz}\right) \quad (2.20)$$

$$Bias(t_8) \approx \theta\bar{Y}\left(C_x^2+C_z^2+C_xC_z\rho_{xz}-C_xC_y\rho_{xy}-C_yC_z\rho_{yz}\right). \quad (2.21)$$

respectively.

(vii) Hanif et al. [6] proposed an estimator which was the modification of the Singh and Espejo (2003) estimator, given as

$$t_9 = \left[\left\{\bar{Y}+k_1\left(\bar{X}-\bar{x}\right)\right\}\left\{k_2\frac{\bar{Z}}{\bar{z}}+\left(1-k_2\right)\frac{\bar{z}}{\bar{Z}}\right\}\right] \quad (2.22)$$

The mean square error and bias expression of t_9 is given by

$$MSE(t_9) = \theta\bar{Y}^2C_y^2\left(1-\rho_{y.xz}^2\right).$$

(2.23) $$Bias(t_9) \approx \theta\bar{Y}C_z^2\left(\frac{1}{4}-\frac{\bar{Z}^2}{\bar{Y}^2}\beta_{yz.x}^2\right). \quad (2.24)$$

where $$\rho_{y.xz}^2 = \frac{\rho_{xy}^2+\rho_{yz}^2-2\rho_{xy}\rho_{xz}}{1-\rho_{xz}^2}$$

$$\beta_{yz.x} = \frac{S_y\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right)}{S_x\left(1-\rho_{xz}^2\right)}.$$

3. PROPOSED ESTIMATOR

In this portion, an estimator has been derived by combining the concept of Bahl and Tuteja [2] exponential type estimator and classical regression estimator. The proposed estimator in the exponential form is given by

$$t_{2G} = \left[K_1\bar{y}+K_2\left(\bar{Z}-\bar{z}\right)\right]\exp\left[\gamma\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]. \quad (3.1)$$

where, K_1, K_2 are real positive constants and γ may take the values -1 and 1.

In order to obtained the bias and mean square error, rewriting (3.1) by using the notations (2.1), we get

$$t_{2G} = \left[K_1\bar{Y}\left(1+\bar{e}_y\right)+K_2\left(\bar{Z}-\bar{Z}\left(1+\bar{e}_z\right)\right)\right]\exp\left[\gamma\frac{\bar{X}-\bar{X}\left(1+\bar{e}_x\right)}{\bar{X}+\bar{X}\left(1+\bar{e}_x\right)}\right] \quad (3.2)$$

After some simplification, (3.2) is given by

$$t_{2G} = \left[K_1\bar{Y}\left(1+\bar{e}_y\right)-K_2\bar{Z}\bar{e}_z\right]\exp\left[\gamma\frac{-\bar{e}_x}{2}\left(1+\frac{\bar{e}_x}{2}\right)^{-1}\right]. \quad (3.3)$$

Expanding the exponential function up to first degree in (3.3), squaring and taking expectations, we may get mean square error as

$$MSE(t_{2G}) \approx \left[\bar{Y}^2\left(\gamma K_1-1\right)^2+\gamma^2\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}\left(1-4H_{yx}\right)\right\} + \theta\gamma^2K_2^2\bar{Z}^2C_z^2+\gamma^2\theta K_1\bar{Y}K_2\bar{Z}C_z^2\left(H_{xz}-2H_{yz}\right)\right] \quad (3.4)$$

In order to get optimum value of K_1 and K_2 we partially differentiating equation (3.4) with respect to K_1 and K_2 and equating to zero we get optimum value of K_1 and K_2.

$$K_1 = \frac{1}{\gamma}\left[\frac{1}{1+\theta C_y^2+\theta\frac{C_x^2}{4}\left(1-4H_{yx}\right)-\theta\frac{C_z^2}{4}\left(H_{xz}-2H_{yz}\right)^2}\right] \quad (3.5)$$

$$K_2 = \frac{-\bar{Y}}{2\gamma\bar{Z}}\left[\frac{\left(H_{xz}-2H_{yz}\right)}{\left[1+\theta C_y^2+\theta\frac{C_x^2}{4}\left(1-4H_{yx}\right)-\theta\frac{C_z^2}{4}\left(H_{xz}-2H_{yz}\right)^2\right]}\right] \quad (3.6)$$

Now putting the value of (3.5) and (3.6) in equation (3.4), and after some simplifications, we get the minimized mean square error as

$$\begin{aligned} MSE \approx \left[\frac{1}{K_1}\right]^2 \Bigg[& \theta\bar{Y}^2C_y^2\left(1+\theta C_y^2\right) \\ & +\frac{\theta}{16}\bar{Y}^2C_z^2\left(H_{xz}-2H_{yz}\right)^2\left\{\theta C_z^2\left(H_{xz}-2H_{yz}\right)^2-4\right\} \\ & +\frac{\theta\bar{Y}^2C_x^2}{4}\left(1-4H_{yx}\right)\left\{1+\frac{\theta C_x^2}{4}\left(1-4H_{yx}\right)\right\} \\ & +\frac{\theta^2C_x^2}{8}\left(1-4H_{yx}\right)\left\{4C_y^2-C_z^2\left(H_{xz}-2H_{yz}\right)^2\right\} \\ & -\frac{\theta^2C_y^2C_z^2}{2}\left(H_{xz}-2H_{yz}\right)^2\Bigg] \end{aligned} \quad (3.7)$$

In order to derive the bias of (3.1) we again use (3.3) and simplify as

$$t_{2Gb} \approx \left[K_1\bar{Y}\left(1+\bar{e}_y\right)-K_2\bar{Z}\bar{e}_z\right]\exp\left[\gamma\frac{-\bar{e}_x}{2}+\gamma\frac{\bar{e}_x^2}{4}\right]. \quad (3.8)$$

Expanding the exponential function up to second degree in (3.8), and after some simplification we get bias as:

$$Bias \approx E\left(t_{2Gb}-\bar{Y}\right) \approx \bar{Y}\left(K_1-1\right)+\frac{\theta\gamma^2K_1\bar{Y}C_x^2}{8}\left(1-\frac{H_{yx}}{2\gamma}\right)+\frac{1}{2}\theta\gamma K_2\bar{Z}H_{xz}C_z^2 \quad (3.9)$$

3.1 Special Cases of Proposed Estimators

(i) For the $\gamma=1$ and $\gamma=-1$, the following regression-cum-exponential ratio type and regression-cum-exponential product type estimator in generalized form may be obtained from proposed estimator

$$t_{2GR} = \left[K_1\bar{y}+K_2\left(\bar{Z}-\bar{z}\right)\right]\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]. \quad (3.10)$$

and

$$t_{2Gp} = \left[K_1\bar{y}+K_2\left(\bar{Z}-\bar{z}\right)\right]\exp\left[\frac{\bar{x}-\bar{X}}{\bar{X}+\bar{x}}\right]. \quad (3.11)$$

Their mean squares are given below:

$$MSE(t_{2GR}) \approx \left[\bar{Y}^2\left(K_1-1\right)^2+\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}\left(1-4H_{yx}\right)\right\} +\theta K_2^2\bar{Z}^2C_z^2+\theta K_1\bar{Y}K_2\bar{Z}C_z^2\left(H_{xz}-2H_{yz}\right)\right] \quad (3.12)$$

and

$$MSE\left(t_{2Gp}\right) \approx \left[\bar{Y}^2\left(K_1-1\right)^2+\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}\left(1+4H_{yx}\right)\right\} +\theta K_2^2\bar{Z}^2C_z^2-\theta K_1\bar{Y}K_2\bar{Z}C_z^2\left(H_{xz}+2H_{yz}\right)\right] \quad (3.13)$$

respectively.

(ii) For the $K_1=1, K_2=b_{yz}, \gamma=1$ and $K_1=1, K_2=b_{yz}, \gamma=-1$, the regression-cum-exponential ratio type and regression-cum-exponential product type estimator may be written as

$$t_{2R} = \left[\bar{y}+b_{yz}\left(\bar{Z}-\bar{z}\right)\right]\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \text{ and}$$

$$t_{2RP} = \left[\bar{y}+b_{yz}\left(\bar{Z}-\bar{z}\right)\right]\exp\left[\frac{\bar{x}-\bar{X}}{\bar{X}+\bar{x}}\right].$$

(iii) For the $K_1=1, K_2=b_{yz}, \gamma=0$, the proposed estimator may take the form of classical regression estimator.

(iv) For the $K_1=1, K_2=0, \gamma=1$, the proposed estimator may take the form of Bhal and Tuteja [1] exponential ratio type estimator

(v) For the $K_1=1, K_2=0, \gamma=-1$, the proposed estimator may take the form of Bhal and Tuteja [1] exponential product type estimator.

4. EFFICIENCY COMPARISONS OF PROPOSED ESTIMATOR OVER OTHER ESTIMATORS

In this section, the theoretical conditions have been derived, when the deduced estimators performs better as compare to some existing estimators. The comparison of t_{2GR} and t_{2GP} is made with t_3, t_4, t_6, t_7, t_8 and t_3, t_5, t_9, respectively, using the mean square errors.

(a) Comparison of $MSE\left(t_{2GR}\right)$ with Classical Regression Estimator

From (3.12) and (2.7), $MSE\left(t_{2GR}\right) < MSE\left(t_3\right)$

$$\left[\bar{Y}^2\left(K_1-1\right)^2+\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}\left(1-4H_{yx}\right)\right\} +\theta K_2^2\bar{Z}^2C_z^2+\theta K_1\bar{Y}K_2\bar{Z}C_z^2\left(H_{xz}-2H_{yz}\right)\right] < \theta\bar{Y}^2C_y^2\left(1-\rho_{xy}^2\right)$$

After simplification we get

$$\bar{Y}^2\left(K_1-1\right)^2+\theta\bar{Y}^2C_y^2\left(K_1^2-1+\rho_{xy}^2\right)+\theta\bar{Y}^2K_1^2\frac{C_x^2}{4}\left(1-4H_{yx}\right) +\theta K_2\bar{Z}C_z^2\left\{K_2+K_1\bar{Y}\left(H_{xz}-2H_{yz}\right)\right\}<0 \quad (4.1)$$

(b) Comparison of $MSE(t_{2GR})$ with Bahl and Tuteja's [1] Estimator

From (3.12) and (2.10), $MSE(t_{2GR}) < MSE(t_4)$

$$\left[\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}(1-4H_{yx})\right\}\right.$$
$$\left.+\theta K_2^2\bar{Z}^2C_z^2+\theta K_1\bar{Y}K_2\bar{Z}C_z^2(H_{xz}-2H_{yz})\right]$$
$$<\theta\bar{Y}^2\left[C_y^2+\frac{C_x^2}{4(1+4H_{yx})}\right]$$

After simplification we get

$$\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2C_y^2(K_1^2-1)+\theta\bar{Y}^2\frac{C_x^2}{4}\left(K_1^2(1-4H_{yx})-\frac{1}{1+4H_{yx}}\right)<0 \quad (4.2)$$

(c) Comparison of $MSE(t_{2GR})$ with Mohanty's [7] Estimator

From (3.12) and (2.15), $MSE(t_{2GR}) < MSE(t_6)$

$$\left[\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}(1-4H_{yx})\right\}\right.$$
$$+\theta K_2\bar{Z}C_z^2\left\{K_2+K_1\bar{Y}(H_{xz}-2H_{yz})\right\}$$
$$<\theta\bar{Y}^2\left[C_y^2(1-\rho_{xy}^2)+C_z^2-2\rho_{yz}C_yC_z+2\rho_{xy}\rho_{xz}C_yC_z\right]$$

After simplification we get

$$\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2C_y^2(K_1^2-1+\rho_{xy}^2)+\theta\bar{Y}^2K_1^2\frac{C_x^2}{4}(1-4H_{yx})$$
$$+C_z^2(\theta K_2^2\bar{Z}^2-1)+\theta K_1\bar{Y}K_2\bar{Z}C_z^2(H_{xz}-2H_{yz})$$
$$2C_yC_z(\rho_{yz}-\rho_{xy}\rho_{xz})<0. \quad (4.3)$$

(d) Comparison of $MSE(t_{2GR})$ with Samiuddin and Hanif [12] Estimator

From (3.13) and (2.18), $MSE(t_{2G}) < MSE(t_7)$

$$\left[\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}(1-4H_{yx})\right\}\right.$$
$$\left.+\theta K_2^2\bar{Z}^2C_z^2+\theta K_1\bar{Y}K_2\bar{Z}C_z^2(H_{xz}-2H_{yz})\right]$$
$$<\theta\bar{Y}^2\left[C_y^2(1-\rho_{xy}^2)+C_z^2(1-\rho_{xz}^2)-2C_yC_x(\rho_{yz}-\rho_{xy}\rho_{xz})\right]$$

After simplification we get

$$\left[\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2C_y^2(K_1^2-1+\rho_{xy}^2)\right.$$
$$+\theta\bar{Y}^2K_1^2\frac{C_x^2}{4}(1-4H_{yx})+C_z^2(\theta K_2^2\bar{Z}^2-1+\rho_{xy}^2)$$
$$\left.+\theta K_1\bar{Y}K_2\bar{Z}C_z^2(H_{xz}-2H_{yz})+2C_yC_x(\rho_{yz}-\rho_{xy}\rho_{xz})\right]<0$$

(4.4)

(e) Comparison of $MSE(t_{2GR})$ with modification of Chand [4] Estimator, which was given Samiuddin and Hanif [12]

From (3.12) and (2.20), $MSE(t_{2GR}) < MSE(t_8)$

$$\left[\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}(1-4H_{yx})\right\}\right.$$
$$\left.+\theta K_2^2\bar{Z}^2C_z^2+\theta K_1\bar{Y}K_2\bar{Z}C_z^2(H_{xz}-2H_{yz})\right]$$
$$<\theta\bar{Y}^2(C_x^2+C_y^2+C_z^2-2C_xC_y\rho_{xy}-2C_yC_z\rho_{yz}+2C_xC_z\rho_{xz})$$

After simplification we get

$$\left[\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2C_y^2(K_1^2-1)+\theta\bar{Y}^2C_x^2\left\{\frac{K_1^2}{4}(1-4H_{yx})-(1-2H_{yx})\right\}\right.$$
$$+\theta C_z^2\left\{K_2^2\bar{Z}^2-\bar{Y}^2(1-2H_{yz})\right\}$$
$$\left.+\theta K_1\bar{Y}K_2\bar{Z}C_z^2(H_{xz}-2H_{yx})-2C_xC_z\rho_{xz}\right]<0$$

(4.5)

(f) Comparison of t_{GP} with Classical Product Estimator

From (3.13) and (2.6), $MSE(t_{2GP}) < MSE(t_3)$

$$\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}(1+4H_{yx})\right\}$$
$$+\theta K_2^2\bar{Z}^2C_z^2-\theta K_1\bar{Y}K_2\bar{Z}C_z^2(H_{xz}+2H_{yz})$$
$$<\bar{Y}^2\theta C_y^2(1-\rho_{xy}^2),$$

After simplification we get

$$\left[\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2C_y^2(K_1^2-1+\rho_{xy}^2)+\theta\bar{Y}^2K_1^2\frac{C_x^2}{4}(1+4H_{yx})\right.$$
$$\left.+\theta K_2^2\bar{Z}^2C_z^2-\theta K_1\bar{Y}K_2\bar{Z}C_z^2(H_{xz}+2H_{yz})\right]<0$$

(4.6)

(g) Comparison of t_{2GP} with Bahl and Tuteja [2] Estimator

From (3.13) and (2.11), $MSE(t_{2GP}) < MSE(t_5)$

$$\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}\left(1+4H_{yx}\right)\right\}$$
$$+\theta K_2^2\bar{Z}^2C_z^2-\theta K_1\bar{Y}K_2\bar{Z}C_z^2\left(H_{xz}+2H_{yz}\right)$$
$$<\bar{Y}^2\theta\left[C_y^2+\frac{C_x^2}{4}\left(1+H_{yx}\right)\right],$$

After simplification we get

$$\left[\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2C_y^2\left(K_1^2-1\right)+\theta\bar{Y}^2\frac{C_x^2}{4}\left(K_1^2\left(1+4H_{yx}\right)-\left(1+H_{yx}\right)\right)\right.$$
$$\left.+\theta K_2^2\bar{Z}^2C_z^2-\theta K_1\bar{Y}K_2\bar{Z}C_z^2\left(H_{xz}+2H_{yz}\right)\right]<0 \quad (4.7)$$

(h) Comparison of t_{GP} with Hanif et al. [6] Estimator

From (3.13) and (2.23),

$$MSE\left(t_{GP}\right)_{\min}<MSE\left(t_9\right)$$
$$\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2K_1^2\left\{C_y^2+\frac{C_x^2}{4}\left(1+4H_{yx}\right)\right\}$$
$$+\theta K_2^2\bar{Z}^2C_z^2-\theta K_1\bar{Y}K_2\bar{Z}C_z^2\left(H_{xz}+2H_{yz}\right)<\theta\bar{Y}^2C_y^2\left(1-\rho_{y.xz}^2\right)$$

After simplification we get

$$\bar{Y}^2(K_1-1)^2+\theta\bar{Y}^2C_y^2\left(K_1^2-1+\rho_{y.xz}^2\right)+\theta\bar{Y}^2K_1^2\frac{C_x^2}{4}\left(1+4H_{yx}\right)$$
$$+\theta K_2^2\bar{Z}^2C_z^2-\theta K_1\bar{Y}K_2\bar{Z}C_z^2\left(H_{xz}+2H_{yz}\right)<0 \quad (4.8)$$

5. NUMERICAL EXAMPLE

In single phase sampling, original data has been used to show the performance of proposed estimator with some existing estimator. The descriptions of the population are given below

Population 1: Anderson [1]
Y: Head length of second son
X: Head length of first son
Z: Head breadth of first son

$N=25, n_1=15$, $\bar{Y}=183.84$, $\bar{X}=185.72$, $\bar{Z}=151.12$, $C_y=0.0546$, $C_x=0.0488$, $C_z=0.0526$, $\rho_{yx}=0.6932$, $\rho_{yz}=0.7108$, $\rho_{xz}=0.7346$

Population II: Cochran [4]
Y: Number of ‘’placebo’’ children
X: Number of paralytic polio cases in the ‘’not inoculated’’ group.
Z: Number of paralytic polio cases in the placebo group

$N=34, n_1=15$, $\bar{Y}=4.92$, $\bar{X}=2.59$, $\bar{Z}=2.91$, $C_y=1.01232$, $C_x=1.23187$, $C_z=1.053516$, $\rho_{yx}=0.7326$, $\rho_{yz}=0.643$, $\rho_{xz}=0.6837$

Population III: Gujrati [8]
Y: The number of wild cats drilled.
X: Price at the well head in the previous period.
Z: Domestic output.

$N=30$, $n_1=12$, $\bar{Y}=10.6374$, $\bar{X}=4.44968$, $\bar{Z}=7.5248$, $C_y=0.21783$, $C_x=0.14732$, $C_z=0.17986$, $\rho_{yx}=-0.4285052$, $\rho_{yz}=0.1377817$, $\rho_{xz}=-0.305424195$.

Table 1 Percentage relative efficiencies for the estimators

Estimators	Populations		
	1	2	3
$\bar{y}$	100	100	100
t_3	192.5025048	215.8441522	*
t_4	172.37096	208.8930029	*
t_6	94.41334467	73.87076248	*
t_7	170.5311765	132.0008015	*
t_8	72.29145849	43.21049749	*
t_{GR}	**230.5157989**	**237.8810639**	*
t_3	*	*	101.935116
t_5	*	*	95.97932128
t_9	*	*	93.87584582
t_{GP}	*	*	**121.6910187**

The results of percent relative efficiencies have been given in table. The percent relative efficiencies have been computed by using the formula $PRE=\frac{Var\left(\bar{Y}\right)}{MSE\left(t_*\right)}\times 100$

The results from population 1-2 have shown that the proposed generalized exponential regression-cum-ratio type estimator is more efficient as compared to classical regression estimator, Mohanty’s estimator [10], exponential ratio-type estimator, Samiuddin and Hanif estimator [15]. The results from population 3 have shown that the proposed generalized form of exponential regression-cum-product type estimator is more efficient as compared to, classical regression type estimator, exponential product-type estimator, and Hanif et.al estimator [9].

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