MODIFIED LAPLACE DECOMPOSITION METHOD AND HOMOTOPY PERTURBATION METHOD FOR LANE-EMDEN TYPE INITIAL VALUE PROBLEMS

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ABSTRACT: *In this paper, we presented a reliable modification of the Laplace Decomposition Method (LDM), and also proposed an effective numerical method Homotopy Perturbation Method (HPM) for second order Lane-Emden type equations. Both methods used for the approximate solutions of nonlinear singular Initial Value Problems (IVPs). He,s polynomials are also used to overcome the difficult calculation. We also proposed the Homotopy Perturbation Method (HPM) is shown to be highly accurate and only a few terms are required to obtain accurate computable solution. The validity of the methods is verified through illustrative examples. The results obtained demonstrate the accuracy and efficiency of the proposed methods. For the accuracy of results we have calculated the equations up to four iterations.*

Keywords: Initial Value Problems, Modified Laplace Decomposition Method, Homotopy Perturbation Method. He,s polynomials

1. INTRODUCTION

Lane-Emden (LE) type IVPs have many applications in mathematics and astrophysics [1, 2]. This problem can be written as:

$$\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}+f(y)=0, \quad (1)$$

subject to $y(0)=A$ and $y'(0)=B$, where A, B are constants and f is a continuous function. This equation is helpful in the study of various models, [1-3]. Moreover, attentions have been paid to LE type problems of the form:

$$\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}+f(x,y)=g(x) \quad 0\le x\le 1 \quad (2)$$

The numerical solution of LE problem is not easy because of the singularity behavior at origin. The approximate solution of the LE equation is given by Adomian decomposition, homotopy perturbation, variational iteration, differential transformation, wavelets-collocation methods [4-12] and so on. Laplace Adomian Decomposition Method proposed in [25, 26] is successfully used to find solutions of the (DE's) [13]. We use this method to obtain exact solutions for nonlinear equations but it generates noise term for inhomogeneous equations [14]. Hussain [15] found a modified method which increases the convergence of the solution as compared with MADM. Here, we use the MLDM to get the exact or approximate solutions of LE type equations.

He,s Homotopy Perturbation Method which is proved to be very effective, simple, and convenient to solve nonlinear boundary value problems. For relatively comprehensive survey on the method and its applications, the reader is referred to He,s review article [16] and monographs [17]. The papers [18-24] constitute a guided tour through the mathematics needed for a proper understanding of Homotopy Perturbation method as applied to various nonlinear problems.

2. MATERIAL AND METHODS

2.1 Modified Laplace Decomposition Method

Here we will briefly discuss the use of MLDM. Simplifying (2) and then taking Laplace transform, we have

$$-s^2L'\{y\}-y(0)+L\{xf(x,y)-xg(x)\}=0,$$

where L is the Laplace transform operator and $L'\{y\}=\frac{dL\{y\}}{ds}$. We decompose $f(x,y)$ into two parts:

$$f(x,y)=R[y(x)]+F[y(x)],$$

where $R[y(x)]$ is a linear and $F[y(x)]$ denotes the nonlinear term. The MLDM gives a solution $y(x)$ by an infinite series as:

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \quad (3)$$

and the nonlinear term takes the form of infinite series of the Adomian polynomials $A_n(x)$ of the form:

$$F\{y(x)\}= \sum_{n=0}^{\infty} A_n(x), \quad (4)$$

where A_n can be given by formula:

$$A_{n=\frac{1}{n!}}\left[\frac{d^n}{d\lambda^n}F(\sum_{n=0}^{\infty}\lambda^n u_n)\right], \quad n = 0, 1, 2, \ldots. \quad (5)$$

Therefore,

$$A_0=F[v_0],$$
$$A_1=v_1F'[v_0],$$
$$A_2=v_2F'[v_0]+\frac{1}{2!}v_1{}^2F''[v_0]$$
$$A_3=v_3F'[v_0]+v_1v_2F''[v_0]+\frac{1}{3!}v_1{}^3F'''[v_0], \quad (6)$$

Using (5) and (6) into (3), we have

$$-S^2L'\{\sum_{n=0}^{\infty}y_n(x)\}-y(0)-L\{xg(x)\}+L\{xR[\sum_{n=0}^{\infty}y_n(x)]+x\sum_{n=0}^{\infty}A_n(x)\}=0$$

or

$$-S^2\sum_{n=0}^{\infty}L'\{y_n(x)\}-y(0)-L\{xg(x)\}+\sum_{n=0}^{\infty}L\{xR(y_n(x))+xA_n(x)\}=0.$$

In general, we have

$$L'\{y_0(x)\}=[-s^2y(0)-s^2L\{xg(x)\}]$$
$$L'\{y_{n+1}(x)\}=[-s^{-2}L\{xR(y_n(x))+xA_n(x)\}] \quad (7)$$

On integration of (7), we obtain

$$L\{y_0(x)\}=\int[-s^{-2}y(0)-s^2L\{xg(x)\}]ds$$
$$L\{y_{n+1}(x)\}=\int[-s^{-2}L\{xR(y_n(x))+xA_n(x)\}]\,ds \quad (8)$$

Taking inverse Laplace transform of (8), we get

$$y_0(x)=L^{-1}\{\int[-s^{-2}y(0)-s^{-2}L\{xg(x)\}]\,ds\}=H(x) \quad (9)$$
$$y_{n+1}(x)= L^{-1}\{\int s^{-2}\, L\{xR(y_n(x))+xA_n(x)\}ds\}$$

Where $H(x)$ arises from the source term and the given initial condition. The choice of (9) as the initial solution produces noise oscillation in iteration process. To overcome this problem, $H(x)$ can be decomposed as $H(x) = H_0(x)+H_1(x)$.

Instead of (9), we suggest $y_0(x)=H_0(x)$,

$$y_0(x) = H_0(x) + L^{-1}\{\int -s^2L\{xR(y_n(x))+xA_n(x)\}\,ds\}$$

and

$$y_{n+1}(x)=L^{-1}[\int s^{-2}L\{xR(y_n(x))+xA_n(x)\}ds]$$

The above solution by MLDM depends on $H_0(x)$and $H_1(x)$.

2.2 He's Homotopy Perturbation Method

This method is very effective in solving non-linear equations. Consider the general non-linear equation $A(u) - f(r) = 0,\ r \in \boldsymbol{\Omega}$ with conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0,\ r \in \Gamma, \tag{10}$$

where A a general differential, B a boundary operator, $f(r)$ a known analytic function and Γ is the boundary Ω.The operator A can be replaced by L and F, where L a linear and F is nonlinear. Therefore $A(u) - f(r) = 0$ takes the form

$$L(u) + F(u) - f(\boldsymbol{r}) = 0, \tag{11}$$

Using homotopy, we define $v(r,p): \Omega \times [0,1] \to R$ as

$$H(v,P) = (1-P)[L(v) - L(u_0)] + P[A(v) - f(r)] = 0,\ P \in [0,1] \tag{12}$$

Or

$$H(v,P) = L(v) - L(u_0) + PL(u_0) + P[F(v) - f(r)] = 0, \tag{13}$$

where $p \in [0,1]$ and u_0 is the initial approximation of $A(u) - f(r) = 0$ satisfy the given conditions. Clearly,

$$H(v,0) = L(v) - L(u_0) = 0 \tag{14}$$

$$H(v,1) = A(v) - f(r) = 0 \tag{15}$$

Change of p from zero to unity is similar to that of $v(r,p)$ from $u_0(r)$ to $u(r)$. This is deformation and $L(v) - L(u_0)$ and $A(v) - f(r)$ are homotopic. If p is a small parameter, the solution of (14) and (15) takes the form:

$$v = v_0 + pv_1 + p^2 v_2 + \ldots \tag{16}$$

For $p = 1$, we have

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \ldots \tag{17}$$

The convergence of (17) is proved in [19]. Implying He's method, the perturbation equation can be built in different ways by homotopy and the initial approximation can easily be chosen. Moreover, homotopy plays important role with required accuracy.

3. NUMERICAL RESULTS

EXAMPLE: 2

Consider

$$\frac{d^2y}{dx^2} + \frac{2}{t}\frac{dy}{dx} + 4\left(2e^y + e^{\frac{y}{2}}\right) = 0, \quad 0 \le t \le 1, \tag{18}$$

Subject to $y(0) = y'(0) = 0$.

3.1 Solution using MLDM:

Simplifying and taking Laplace transform, we write

$$\left\{-s^2 \frac{d}{ds} y(s) - 2sy(s) - y(0)\right\} + 2sy(s) - 2y(0) = -L\left\{4t\left(2e^y + e^{\frac{y}{2}}\right)\right\}. \tag{19}$$

Put $y(t) = \sum_{n=0}^{\infty} y_n(t)$ and $F\{y(t)\} = \sum_{n=0}^{\infty} A_n(t)$ in (19), we have

$-s^2 L'\left\{\sum_{n=0}^{\infty} y_{n(t)}\right\} - y(0) + L\{4t(\sum_{n=0}^{\infty} A_n)\} = 0$

$-s^2 \sum_{n=0}^{\infty} L'\{y_n(t)\} - y(0) + \sum_{n=0}^{\infty} L\{4t(A_n)\} = 0.$

The recursive relation is obtained as

$$L'\{y_0\} = -s^{-2} y(0) = 0 \tag{20}$$

$$L'\{y_n\} = s^{-2} L\{4tA_{n-1}\}$$

Where $F(y)$ can be decomposed as $F[y] = 2e^y + e^{\frac{y}{2}}$ So the few Adomian Polynomial are

$$A_0 = 2e^{y_0} + e^{\frac{y_0}{2}} = 3,$$

$$A_1 = y_1\left(2e^{y_0} + \frac{1}{2}e^{\frac{y_0}{2}}\right) = y_1\left(\frac{5}{2}\right)$$

$$A_2 = y_2\left(e^{y_0} + \frac{1}{2}e^{\frac{y_0}{2}}\right) + \frac{{y_1}^2}{2!}\left(2e^{y_0} + \frac{1}{4}e^{\frac{y_0}{2}}\right) = y_2\left(\frac{5}{2}\right) + \frac{{y_1}^2}{2!}\left(\frac{9}{4}\right), \tag{21}$$

$$A_3 = y_3\left(2e^{y_0} + \frac{1}{2}e^{\frac{y_0}{2}}\right) + y_1 y_2\left(2e^{y_0} + \frac{1}{4}e^{\frac{y_0}{2}}\right) + \frac{{y_1}^3}{3!}\left(2e^{y_0} + \frac{1}{8}e^{\frac{y_0}{2}}\right),$$

Integrating (20) and then Taking the inverse Laplace transform we get

$$y_0(t) = 0 \tag{22}$$

$$y_n(t) = L^{-1}[\int\{S^{-2}L(4tA_{n-1})\}ds]$$

Using initial conditions and (21) in above equation , we get these results

$$y_0 = 0$$
$$y_1 = -2t^2$$
$$y_2 = t^4$$
$$y_3 = -\frac{2}{3}t^6$$
$$y_4 = \frac{1}{2}t^8$$

Thus the solution of problem becomes.

$$y(t) = -2\left(t^2 - \frac{1}{2}t^4 + \frac{1}{3}t^6 - \frac{1}{4}t^8 + \cdots\right)$$

3.2 Solution using HPM.

With equation (19) we construct the following homotopy.

$$-s^2 L'\{y\} - y(0) + PL\left\{4t\left(2e^y + e^{\frac{y}{2}}\right)\right\} = 0 \tag{23}$$

where $F(y) = 2e^y + e^{\frac{y}{2}}$, and $p \in [0,1]$

In view of He's HPM , we assume the solution of given equation as

$$y = y_0 + Py_1 + P^2 y_2 + \ldots.. \tag{24}$$

The term $F(y)$ is such that

$$F(y) = H(y_0) + PH(y_0, y_1) + P^2 H(y_0, y_1, y_2) + \cdots \tag{25}$$

where

$$H(y_0, y_1, \ldots y_n) = \frac{1}{n!}\frac{d^n}{dP^n} F\left(\sum_{k=0}^{n} P^k y_k\right)_{P=0}$$

Using (24) and (25) into (23)

$$-S^2 L'\{y_0 + Py_1 + P^2 y_2 + \ldots\} - y(0) + PL\{4t(H(y_0) + PH(y_0, y_1) + P^2 H(y_0, y_1, y_2) + \ldots)\} = 0$$

This implies that

$$[-s^2 L'(y_0) - y(0)]P^0 + [-s^2 L'(y_1) + L\{4tH(y_0)\}]P^1 + [-s^2 L'(y_2) + L\{4tH(y_0, y_1)\}]P^2 + [-s^2 L'(y_3) + L\{4tH(y_0, y_1, y_2)\}]P^3 + \ldots = 0 \tag{26}$$

We find out the He,s polynomial same as (21).Integrating (23) and then Taking the inverse Laplace transform Thus we have

$$y_0(t) = 0 \tag{27}$$

$$y_n(t) = L^{-1}[\int\{S^{-2}L(4tH_{n-1})\}ds]$$

Using He,s polynomial and (27) in (26), we have

$$y_0 = 0$$
$$y_1 = -2t^2$$
$$y_2 = t^4$$
$$y_3 = -\frac{2}{3}t^6$$
$$y_4 = L^{-1}[\int\{S^{-2}L(4tH_3)\}ds]$$

$y_4 = \frac{1}{2}t^8$ and so on.

Therefore, we write

$$y(t) = -2\left(t^2 - \frac{1}{2}t^4 + \frac{1}{3}t^6 - \frac{1}{4}t^8 + \cdots\right).$$

Summary of results and their graphical representation is given as follows.

Table -1: Summary of results obtained in Example 1.

T	Exact	Approximate	Error
0.1	-0.019900661706336	-0.019900661706336	0.0000000000E+00
0.15	-0.044501217869639	-0.044501217869640	9.9920072216E-16
0.2	-0.078441426306563	-0.078441426306565	1.9984014443E-15
0.25	-0.121249243632870	-0.121249243633042	1.7200130209E-13
0.3	-0.172355392482105	-0.172355392488551	6.4460103921E-12
0.35	-0.231116681268545	-0.231116681405498	1.3695297674E-10
0.4	-0.296840010236547	-0.296840012156592	1.9200450119E-09
0.45	-0.368805445995559	-0.368805465583314	1.9587755029E-08
0.5	-0.446287102628420	-0.446287258087643	1.5545922305E-07
0.55	-0.528570989290770	-0.528571996100404	1.0068096340E-06
0.6	-0.614969399495921	-0.614974911671276	5.5121753550E-06
0.65	-0.704831774036698	-0.704857981377831	2.6207341133E-05
0.7	-0.797552239914736	-0.797662735858658	1.1049594392E-04
0.75	-0.892574205256839	-0.892994275926113	4.2007066927E-04
0.8	-0.989392483672214	-0.990851939806924	1.4594561347E-03
0.8	-0.989392483672214	-0.990851939806924	1.4594561347E-03
0.85	-1.087553447811350	-1.092238756153360	4.6853083420E-03
0.9	-1.186653690555460	-1.200679715048060	1.4026024493E-02
0.95	-1.286337621507400	-1.325793430948660	3.9455809441E-02
1	-1.386294361119890	-1.491269841269840	1.0497548015E-01

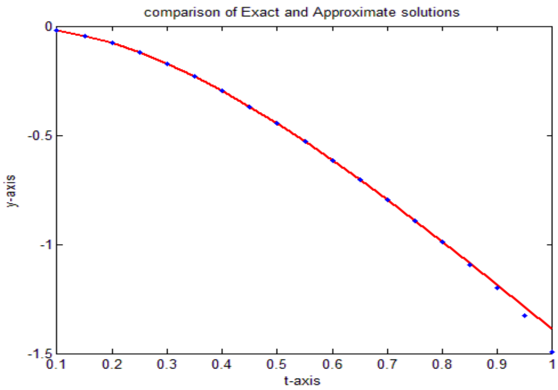

Figure 1: Graphical comparison of Exact and Approximate Solutions of Example 1

EXAMPLE: 2

Consider

$$\frac{d^2y}{dx^2}+\frac{2}{t}\frac{dy}{dx}-6y=4y\ln y\ ,0\le t\le 1 \qquad (28)$$

subject to $y(0)=1$ and $y'(0)=0$.

3.3. Solution using MLDM

Multiplying (28) by t, taking Laplace transform we have

$$-s^2L'\{y\}=L\{6yt+4yt\ln y\} \qquad (29)$$

Put $y(t)=\sum_{n=0}^{\infty}y_n(t)$ and $F\{y(t)\}=\sum_{n=0}^{\infty}A_n(t)$

$$-s^2L'\{\textstyle\sum_{n=0}^{\infty}y_n(t)\}=L\{6t\sum_{n=0}^{\infty}y_n(t)+4t\sum_{n=0}^{\infty}A_n\ln y\}$$

$$-s^2\textstyle\sum_{n=0}^{\infty}L'\{y_n(t)\}=\sum_{n=0}^{\infty}L\{6ty_n(t)+4tA_n\ln y\}$$

Hence we have the recursive relation

$$L'\{y_0\}=-s^{-2}y(0) \qquad (30)$$

$$L'\{y_n\}=-s^{-2}L\{6y_{n-1}t+4A_{n-1}t\ln y\}$$

where the nonlinear operator $F[y]=y\ln y$ is decomposed as in [4] in terms of the Adomian Polynomials for $F[y]=y\ln y$ are computed as follows;

$$A_0=F[y_0]=y_0\ln y_0$$

$$A_1=y_1F'[y_0]=y_1(1+\ln y_0)$$

$$A_2=y_2F'[y_0]+\frac{1}{2!}y_1^2F''[y_0]=y_2(1+\ln y_0)+\frac{1}{2!}y_1^2\left(\frac{1}{y_0}\right)$$

$$A_3=y_3F'[y_0]+y_1y_2F''[y_0]+\frac{1}{3!}y_1^3F'''[y_0]=$$

$$y_3(1+\ln y_0)+y_1y_2\left(\frac{1}{y_0}\right)+\frac{1}{3!}y_1^3\left(-\frac{1}{y_0^2}\right)$$

Integrating both sides of eq (30) and then taking the inverse Laplace transform, we have

$$y_0(t)=L^{-1}\{\textstyle\int -s^{-2}ds\} \qquad (31)$$

$$y_n(t)=L^{-1}[\textstyle\int -s^{-2}L\{6y_{n-1}t+4A_{n-1}t\}ds]$$

Using $y(0)=1$, $y'(0)=0$,in (31) we have

$$y_0=L^{-1}\left(-\frac{s^{-2+1}}{-2+1}\right)$$

$$y_0=1$$

$$y_1=L^{-1}[-s^{-2}L\{6y_0t+4A_0t\}ds]$$

$$y_1=t^2$$

$$y_2=L^{-1}[\textstyle\int -s^{-2}L\{6y_1t+4A_1t\}ds]$$

$$y_2=\frac{1}{2}t^4$$

$$y_3=L^{-1}[\textstyle\int -s^{-2}L\{6y_2t+4tA_2\}\,ds]$$

$$y_3=L^{-1}\left[\frac{7\times5!\times6}{7\times6}.\frac{1}{s^7}\right]$$

$$y_3=\frac{1}{3!}t^6$$

$$y_4=L^{-1}[\textstyle\int\{-s^{-2}L(6y_3t+4tA_3)\}ds]$$

$$y_4=L^{-1}\left[\frac{1}{3\times8}.\frac{8!}{s^{8+1}}\right]$$

$$y_4=\frac{1}{4!}t^8$$

Hence the solution is given blow in the series form

$$y(t)=1+t^2+\frac{1}{2!}t^4+\frac{1}{3!}t^6+\frac{1}{4!}t^8+\ldots$$

The exact solution is $y(t)=e^{t^2}$

3.4 Solution using HPM

Using equation (29) , we construct the following homotopy

$$-s^2L'(y)-PL\{6ty+4ty\ln y\}=0, \qquad (32)$$

Where $F(y)=y\ln y\,,p\in[0,1]$ is the embedding parameter? We consider solution of (32) as:

$$y=y_0+Py_1+P^2y_2+\ldots.. \qquad (33)$$

and

$$F(y)=H(y_0)+PH(y_0,y_1)+P^2H(y_0,y_1,y_2)+\ldots, \qquad (34)$$

where

$$H(y_0,y_1,\ldots y_n)=\frac{1}{n!}\frac{d^n}{dP^n}F(\textstyle\sum_{k=0}^{n}P^ky_k)_{P=0}$$

From (32), (33) and (34) and comparing the coefficients, we have

$$[-s^2L'(y_0)]P^0+[-s^2L'(y_1)-L\{6ty_0+4tH(y_0)\}]P^1+$$
$$[-s^2L'(y_2)-L\{6ty_1+4tH(y_0,y_1)\}]P^2+$$
$$[-s^2L'(y_3)-L\{6ty_2+4tH(y_0,y_1,y_2)\}]P^3+\ldots=0 \qquad (35)$$

Thus

$$H_0=H(y_0)=y_0\ln y_0,$$

$$H_1=H(y_0,y_1)=y_1(1+\ln y_0)$$

$$H_2=H(y_0,y_1,y_2)=y_2(1+\ln y_0)+\frac{y_1^2}{2!}\left(\frac{1}{y_0}\right), \qquad (36)$$

$$H_3=H(y_0,y_1,y_2,y_3)=y_3(1+\ln y_0)+y_1y_2\left(\frac{1}{y_0}\right)+\frac{y_1^3}{3!}\left(-\frac{1}{y_0^2}\right)$$

Integrating (29), and then Applying inverse Laplace transform , we obtain

$$y_0(t)=L^{-1}\{\textstyle\int -s^{-2}ds\} \qquad (37)$$

$$y_n(t)=L^{-1}[\textstyle\int s^{-2}L\{6y_{n-1}t+4A_{n-1}t\}ds]$$

From (35), (36) and (37), we get

$$y_0=L^{-1}\left(-\frac{s^{-2+1}}{-2+1}\right)$$

$$y_0=1$$

$$y_1=L^{-1}[-s^{-2}L\{6y_0t+4A_0t\}ds]$$

$$y_1=t^2$$

$$y_2=L^{-1}[\textstyle\int -s^{-2}L\{6y_1t+4A_1t\}ds]$$

$$y_2=\frac{1}{2}t^4$$

$$y_3=L^{-1}[\textstyle\int -s^{-2}L\{6y_2t+4tA_2\}\,ds]$$

$$y_3=\frac{1}{3!}t^6$$

$$y_4=L^{-1}[\textstyle\int\{-s^{-2}L(6y_3t+4tA_3)\}ds]$$

$$y_4=\frac{1}{4!}t^8$$

$$y(t)=1+t^2+\frac{1}{2!}t^4+\frac{1}{3!}t^6+\frac{1}{4!}t^8+\cdots$$

Summary of results and their graphical representation is given as follow.

Table-2: Summary of results obtained in example 2.

T	Exact	Approximate	Error
0	1.000000000000000	1.000000000000000	0
0.05	1.002503127605795	1.002503127605794	0.000000000000001
0.1	1.010050167084168	1.010050167083334	0.000000000000834
0.15	1.022755034164446	1.022755034116211	0.000000000048235
0.2	1.040810774192388	1.040810773333333	0.000000000859055
0.25	1.064494458917860	1.064494450887044	0.000000008030815
0.25	1.064494458917860	1.064494450887044	0.000000008030815
0.3	1.094174283705210	1.094174233750000	0.000000049955210
0.35	1.130319120074011	1.130318885418294	0.000000234655717
0.4	1.173510870991810	1.173509973333333	0.000000897658477
0.45	1.224460085121915	1.224457148959961	0.000002936161954
0.5	1.284025416687741	1.284016927083334	0.000008489604408
0.55	1.353237676421172	1.353215456512044	0.000022219909128
0.6	1.433329414560340	1.433275840000000	0.000053574560340
0.65	1.525771219603461	1.525650626824544	0.000120592778917
0.7	1.632316219955379	1.632060167083334	0.000256052872045
0.75	1.755054656960299	1.754537582397461	0.000517074562838
0.8	1.896480879304952	1.895481173333334	0.000999705971618
0.85	2.059575719127713	2.057715149480794	0.001860569646918
0.9	2.247907986676472	2.244559633750000	0.003348352926472
0.95	2.465759811603786	2.459910958074544	0.005848853529242
1	2.718281828459046	2.708333333333333	0.009948495125712

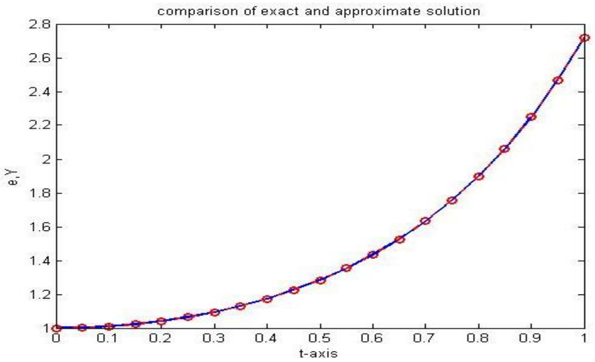

Figure 2: Graphical comparison of Exact and Approximate Solutions of Example 2.

4. CONCLUSION

In this paper we have successfully employed modified Laplace Decomposition Method (MLDM) for following non-linear singular initial value problems. It is demonstrated that the presented approach can accelerate the rapid convergence of series solution when compared with other methods. It is shown that MLDM is simple and easy to use and produced reliable results. We applied a different implementation of the Homotopy Perturbation Method deduced from He's HPM, it is clear that HPM for provides us with a freedom of choose for construction of Homotopy. Choice of Homotopy for the problem plays a significant role foe the accuracy of solution. He's HPM overcome the complex and problematic calculation of problems. After obtaining the approximate solutions of higher order non-linear initial value problems, the results proved that HPM is a powerful tool for the solution of singular IVP's, and onlt other hand MLDM is a little complicated as compare to HPM. The comparison of these two methods shows that results are almost equivalent.

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