VERTEX EQUITABLE LABELING OF SUPER SUBDIVISION GRAPHS

P.Jeyanthi,
Research Centre, Department of Mathematics,Govindammal Aditanar College for Women,
Tiruchendur-628 215, Tamilnadu,India
e.mail: jeyajeyanthi@rediffmail.com
A. Maheswari,
Department of Mathematics, Kamaraj College of Engineering and Technology,Virudhunagar,Tamilnadu, India
e.mail:bala_nithin@yahoo.co.in
M.Vijayalakshmi
Department of Mathematics,Dr.G.U. Pope College of Engineering,Sawyerpuram,Thoothukudi District,
Tamil Nadu, India.
Email:viji_mac@rediffmail.com

ABSTRACT: *Let G be a graph with p vertices and q edges and* $A = \{0, 1, 2,..., \lceil \frac{q}{2} \rceil\}$ *A vertex labeling* $f: V(G) \to A$ *induces. an edge labeling* f^* *defined by* $f^*(uv) = f(u) + f(v)$ *for all edges uv. For* $a \in A$, *let* $v_f(a)$ *be the number of vertices v with* $f(v) = a$. *A graph G is said to be vertex equitable if there exists a vertex labeling f such that for all a and b in A,* $|v_f(a) - v_f(b)| \le 1$ *and the induced edge labels are 1, 2, 3,…, q. In this paper, we prove that the graphs* $S^*(P_n \odot K_1)$, $S^*(B(n,n))$, $S^*(P_n \times P_2)$ *and* $S^*(Q_n)$ *of quadrilateral snake are vertex equitable.*

Key words: Vertex equitable labeling, vertex equitable graph
AMS Classification (2010): 05C78

1. INTRODUCTION:

All graphs considered here are simple, finite, connected and undirected .We follow the basic notations and terminologies of graph theory as in [1]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and a detailed survey of graph labeling can be found in [2]. The vertex set and the edge set of a graph are denoted by $V(G)$ and $E(G)$ respectively. The concept of vertex equitable labeling was due to Lourdusamy and Seenivasan in [3] and further studied in [4,5,6,7,8,9]. Let G be a graph with p vertices and q edges and $A=\{0,1,2, ..., \lceil \frac{q}{2} \rceil\}$. A graph G is said to be vertex equitable if there exists a vertex labeling $f: V(G)\to A$ that induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that $|v_f(a) - v_f(b)| \le 1$ for all a and b in A, and the induced edge labels are 1, 2, 3,…, q, where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$. The vertex labeling f is known as vertex equitable labeling. A graph G is said to be a vertex equitable if it admits vertex equitable labeling. The corona $G_1 \odot G_2$ of the graphs G_1 and G_2 is defined as a graph obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2. A cartesian product of two graphs G_1 and G_2 is the graph $G_1 \times G_2$ such that its vertex set is a cartesian product of $V(G_1)$ and $V(G_2)$ i.e. $V(G_1 \times G_2) = V(G_1) \times V(G_2) = \{(x, y) / x \in V(G_1), y \in V(G_2)\}$ and its edge set is defined as $E(G_1 \times G_2) = \{((x_1, x_2), (y_1, y_2)) / x_1 = y_1$ and $(x_2, y_2) \in E(G_2)$ or $x_2 = y_2$ and $(x_1, y_1) \in E(G_1)\}$. The comb $P_n \odot K_1$ is a graph obtained by joining a single pendant edge to each vertex of a path P_n .The bistar $B_{m,n}$ is a graph obtained from K_2 by joining m pendant edges to one end of K_2 and n pendant edges to the other end of K_2. The graph $P_n \times P_2$ is called a ladder graph. The quadrilateral snake $D(Q_n)$ is a graph obtained from a path P_n with vertices $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to the new vertices v_i, x_i respectively and then joining v_i, x_i for i=1,2,…,n-1. Let G be a graph. The super subdivision graph $S^*(G)$ is obtained from G by replacing every edge e of G by a complete bipartite graph $K_{2,m} (m \ge 2)$ in such a way that the ends of e are merged with the two vertices of the 2-vertices part of $K_{2,m}$ after removing the edge e from G.

We use the following known results in the subsequent theorems.

Theorem 1.1 [9] Let $G_1(p_1,q_1)$, $G_2(p_2,q_2)$,…, $G_m(p_m,q_m)$ be a vertex equitable graphs with q_i's are even(i=1,2,…,m) and u_i, v_i be the vertices of G_i $(1 \le i \le m)$ labeled by 0 and $\frac{q_i}{2}$ .Then the graph G obtained by identifying v_1 with u_2 and v_2 with u_3 and v_3 with u_4 and so on until we identify v_{m-1} with u_m is a vertex equitable graph.

Theorem 1.2 [3] The super subdivision graph C_n is vertex equitable if $n \equiv 0$ or 3(mod4) ,.

2. MAIN RESULTS

Theorem 2.1: The super subdivision graph $S^*(P_n \odot K_1)$ is a vertex equitable graph.

Proof: Let $v_1, v_2, ..., v_n, u_1, u_2, ..., u_n$ be the vertices of the comb graph $P_n \odot K_1$ and $v_i u_i$ $(1 \le i \le n)$, $v_i v_{i+1}$ $(1 \le i \le n-1)$ be the edges of $P_n \odot K_1$. Let u_{ij}, v_{ij} be the vertices of m vertices part, where $1 \le i \le n, 1 \le j \le m$. Clearly $S^*(P_n \odot K_1)$ has $2n+mn+(n-1)m$ vertices and $2m(2n-1)$ edges. Let $A=\{0,1,2,\ldots,m(2n-1)\}$. Define a vertex labeling $f: V(S^*(P_n \odot K_1)) \to A$ as follows:

For $1 \le i \le n$, $f(v_i) = \begin{cases} m(2i-1) & \text{if } i \text{ is odd} \\ 2m(i-1) & \text{if } i \text{ is even} \end{cases}$

$f(u_i) = \begin{cases} 2m(i-1) & \text{if } i \text{ is odd} \\ m(2i-1) & \text{if } i \text{ is even}, \end{cases}$

For $1 \le i \le n, 1 \le j \le m$,

$f(v_{ij}) = \begin{cases} m(2i-1)+j & \text{if } i \text{ is odd} \\ m2i+j & \text{if } i \text{ is even}. \end{cases}$

For $2 \le i \le n$, $1 \le j \le m$ $f(u_{1j}) = j$ and

$f(u_{ij}) = \begin{cases} 2(i-1)m+j & \text{if } i \text{ is odd} \\ 2(i-2)m++m+j & \text{if } i \text{ is even}. \end{cases}$ It can be verified that the induced edge labels of $S^*(P_n \odot K_1)$ are $1, 2, \ldots, 2m(2n-1)$ and $|v_f(i) - v_f(j)| \le 1$ for $i, j \in A$. Hence $S^*(P_n \odot K_1)$ is a vertex equitable graph

Example 1. The vertex equitable labeling of the super subdivision graph, $S^*(P_2 \odot K_1)$ is given in Figure 1.

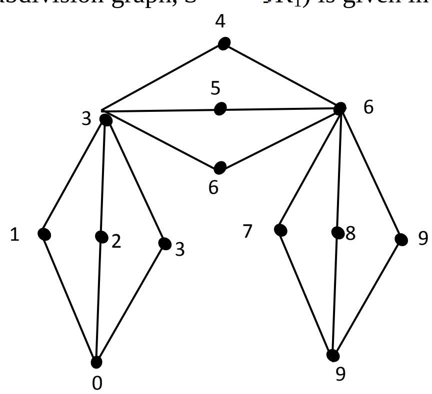

Figure 1: Vertex equitable labeling of $S^*(P_2 \odot K_1)$

Theorem 2.2. The super subdivision graph $S^*(B(n,n))$ is a vertex equitable graph.

Proof: Let $v_1, v_2, ..., v_n, u_1, u_2, ..., u_n, u, v$ be the vertices of the bistar $B(n,n)$ and uv, vv_i, uu_i $(1 \le i \le n)$ be the edges of $B(n,n)$. Let u_{ij}, v_{ij}, c_j be the vertices of m vertices part $1 \le i \le n$, $1 \le j \le m$. Clearly the graph $S^*(B(n,n))$ has $2nm+m+2n+2$ vertices and $2m(2n+1)$ edges. Let $A=\{0,1,2,\ldots,m(2n+1)\}$.

Define a vertex labeling $f: V(S^*(B(n,n))) \to A$ as follows: $f(v)=m$, $f(u)=2nm$.

For $1 \le i \le n$ $f(u_i) = m(2i+1)$, $f(v_i) = 2m(i-1)$.

For $2 \le i \le n$, $1 \le j \le m$ $f(v_{1j}) = j$,

$f(v_{ij}) = 2m(i-1)-(j-1)$. For $1 \le i \le n$, $1 \le j \le m$ $f(u_{ij}) = m(2i+1)-(j-1)$ and $f(c_j) = 2mn-(j-1)$.

It can be verified that the induced edge labels of $S^*(B(n,n))$ are $1, 2, \ldots, 2m(2n+1)$ and $|v_f(i) - v_f(j)| \le 1$ for $i, j \in A$. Hence $S^*(B(n,n))$ is a vertex equitable graph.

Example 2. The vertex equitable labeling of the super subdivision graph $S^*(B(2,2))$ is given in Figure 2.

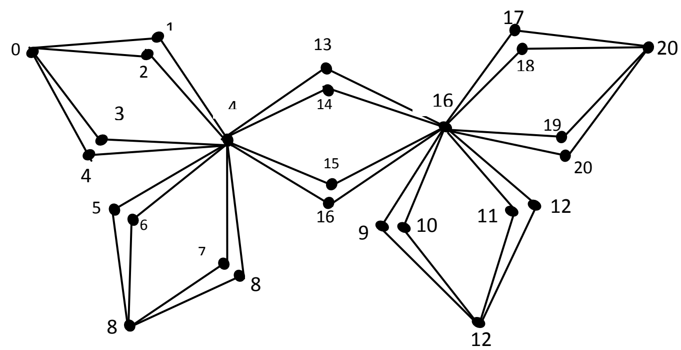

Figure 2: Vertex equitable labeling of $S^*(B(2,2))$

Theorem 2.3 The super subdivision graph $S^*(P_n \times P_2)$ is a vertex equitable graph.

Proof: Let $v_1, v_2, ..., v_n, u_1, u_2, ..., u_n$ be the vertices of the ladder graph $P_n \times P_2$ and $v_i u_i$ $(1 \le i \le n)$, $u_i u_{i+1}, v_i v_{i+1}$ $(1 \le i \le n-1)$ be the edges of $P_n \times P_2$ ..Let u_{ij}, v_{ij}, c_{ij} be the vertices of m vertices part, where $1 \le i \le n, 1 \le j \le m$. Clearly $S^*(P_n \times P_2)$ has $2n(m+1)-m$ vertices and $6mn-4m$ edges. Let $A=\{0,1,2,\ldots,3mn-2m)\}$. Define a vertex labeling $f: V(S^*(P_n \times P_2)) \to A$ as follows:

For $1 \le i \le \left[\frac{n}{2}\right]$, $f(u_{2i-1}) = 6m(i-1)$, $f(v_{2i-1}) = m(6i-5)$

For $1 \le i \le \left[\frac{n}{2}\right]$, $f(u_{2i}) = 2m(3i-1)$, $f(v_{2i}) = 3m(2i-1)$

For $1 \le i \le \left[\frac{n}{2}\right], 1 \le j \le m$, $f(u_{2i-1,j}) = 2m(3i-1)-j+1$, $f(v_{2i-1,j}) = m(6i-5)+j$

For $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor, 1 \le j \le m,$ $f(u_{2i,j}) = m(6i-1) - j + 1$, $f(v_{2i,j}) = m(6i+1) - j + 1$

For $1 \le j \le m,$

$$f(c_{1,j}) = j,\ f(c_{2i-1,j}) = 6m(i-1) - j + 1 \text{ if } 2 \le i \le \left\lceil \frac{n}{2} \right\rceil,$$

$$f(c_{2i,j}) = 3m(2i-1) - j + 1 \text{ if } 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor.$$

It can be verified that the induced edge labels of $S^*(P_n \times P_2)$ are 1, 2,…, $6mn$-$4m$ and $|v_f(i) - v_f(j)| \le 1$ for $i, j \in A$. Hence $S^*(P_n \times P_2)$ is a vertex equitable graph.

Example 3. The vertex equitable labeling of the super subdivision graph, $S^*(P_4 \times P_2)$ is given in Figure3.

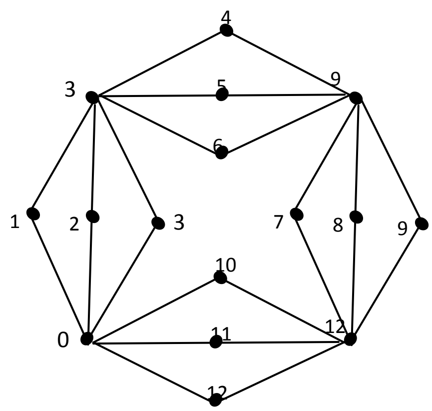

Figure 3: vertex equitable labeling of $S^*(P_2 \times P_2)$

Theorem 2.4 The **s**uper subdivision graph $S^*(Q_n)$ is a vertex equitable graph

Proof: By Theorem 1.2, $S^*(Q_2)$ is a vertex equitable graph. Let $G_i = S^*(Q_2)$, $1 \le i \le n-1$ and u_i, v_i be the vertices with labels 0 and $\frac{q_i}{2}$ respectively. By Theorem 1.1, the graph $S^*(Q_n)$ admits vertex equitable labeling.

Example 2.8 The vertex equitable labeling of $S^*(Q_3)$ is given in Figure 4.

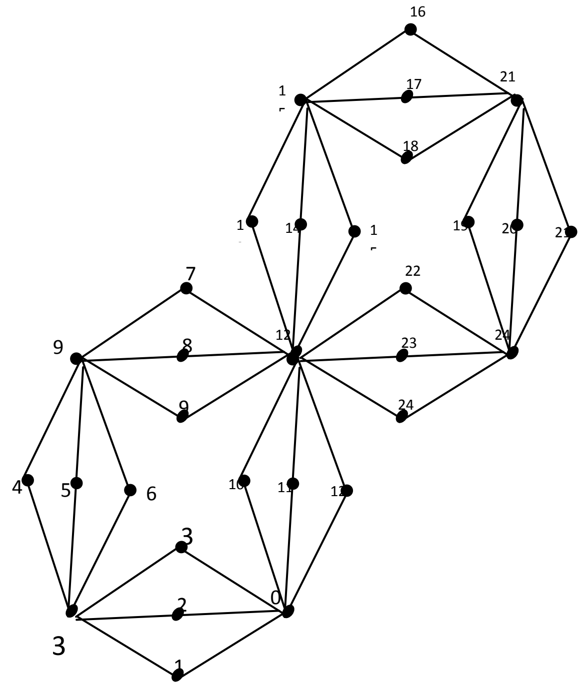

Figure 4:vertex equitable labeling of $S^*(Q_3)$

REFERENCES

[1].Harary.F, Graph theory, Addison Wesley, Massachusetts, (1972).

[2]. Gallian J.A , A Dynamic Survey of graph labeling, *The Electronic Journal of Combinatorics*, #DS6, 2014.

[3].Lourdusamy .A and Seenivasan. M , Vertex equitable labeling of graphs, *Journal of Discrete Mathematical Sciences &Cryptography*,11(6):727-735(2008).

[4]. Jeyanthi .P and Maheswari .A, Some Results on Vertex Equitable Labeling, O*pen Journal of Discrete Mathematics*, 2): 51-57 (2012).

[5]. Jeyanthi .P and Maheswari .A, Vertex equitable labeling of Transformed Trees, J*ournal of Algorithms and Computation* 44 :9 -20 (2013).

[6] . Jeyanthi .P and Maheswari .A, Vertex equitable labeling of cyclic snakes and bistar graphs, *Journal of Scientific Research*, Vol. 6,No.1(2014),79-85.

[7]. Jeyanthi .P and Maheswari .A, Vertex equitable labeling of cycle and path related graphs,*Utilitas Mathematica*, (to appear)..

[8] Jeyanthi .P and Maheswari .A and Vijayalakshmi.M, Vertex equitable labeling of double alternate snake graphs, (preprint).

[9] . Jeyanthi .P and Maheswari .A, and Vijayalakshmi.M **,** New results on vertex equitable labeling, (preprint).