ON SOME PERTURBED SERIES OF NEUTRAL STRONTIUM

Rizwana Siddique, *Salman Raza, Naveed Ali, .Zaheer Uddin
Department of Physics, University of Karachi, Karachi, Postal code 75270, Pakistan.
Emails: rizwanasiddiq@gmail.com, *salmanraza1928@gmail.com, naveedalirajput@gmail.com, zaheer103@yahoo.com.

ABSTRACT: *Rydberg energies can be calculated precisely by the semi-empirical method given by Zhang et al. The method works well for unperturbed spectral series. A problem arises when one or more perturbers exist. In such cases, the procedure is not simple until and unless one finds the energy of the perturbers. Neutral Strontium has some perturbed series; we present theoretical calculations of spectral energy levels for the* ***[Kr]5snl***$_{(l=s,\ p,\ d,\ f)}$ ($^1S_0$,$^3S_1$,$^1P^o_1$, $^1D_2$,$^3D_{(1,2,3)}$,$^1F^o_3$,$^3F^o_{(2,3,4)}$) *perturbed and unperturbed series and radiative lifetimes for the* ***[Kr]5snl***$_{(l=s,\ p,\ d,\ f)}$ ($^3S_1$,$^1P^o_1$,$^1D_2$,$^3D_{(2,3)}$,$^1F^o_{(3,4,5)}$) *series of neutral strontium (Sr-I). Out of eleven spectral series five were perturbed. Our results show good agreement with experimental results. The deviation in spectral energies is less than 0.1 cm^{-1} for most cases. The spectral energy levels series and radiative lifetimes up to principal quantum numbers* ***n=50*** *are presented.*

Key Words: Spectral energy levels, even/odd parity, weakest bound electron, foreign level perturbation, Rydberg atoms, Radiative lifetimes.

1. INTRODUCTION:

Spectral energy level data of Rydberg atoms is of great practical importance in the understanding of the fabrication of heavy atoms in the galaxy, the opacity of lines emitted in all ranges, fusion research, stellar nucleosynthesis in stars, environmental studies for absorption spectroscopy, and in the exploration of chemical properties of the astrophysical bodies. Many experimental and theoretical techniques have been developed for spectral analysis of Rydberg atoms [1-10]. Theoretical techniques developed in the past that studied Rydberg atoms had onerous calculations, especially when considering many valence electron systems. The difficulty was solved by Zheng et al. [11] by putting forward a semi-empirical model called the Weakest Bound Electron Potential Model Theory (WBEPMT). The model considers the successive ionization of atoms and molecules, by separating the atom into an ion core and the electron outside the core as a weakest bound electron (WBE). Many theoretical types of research were conducted based on this concept and results show good agreement between theoretical and experimental work [12-23]. Poirier et al. used perturbation theory by utilizing Coulomb's Green function with the zero-order potential model, to calculate the energy of *4dnl (l=s, d, g)* Rydberg states of neutral strontium. In addition, polarization effects were studied in the auto-ionization process of alkaline earth metals [24, 25]. In 1978, Cooke et al performed experimental research on doubly excited state of Sr-I [26]. Similarly, Bi-ru et al. performed an experimental investigation on doubly excited (*6snl, l=s, d*) auto-ionizing states of Sr-I by utilizing the three-step photo ionization excitation process [27]. In 1994 Griesmann et al. experimentally measured the total photo-absorption cross-section for doubly excited states of Sr-I, in the limit of *5pns* [28]. Aymar et al. [29] and Kompitsas et al. [30, 31] utilized multichannel quantum defect theory (MQDT). Kompitsas et al performed experimental and theoretical calculations below the threshold value of Sr-I for J= 1, 2 [32]. In 2001, Cohen et.al investigated doubly excited autoionizing states of Sr-I for the even parity 5pnp J = 0, 1, 2 [33]. In 2007 Zhang et al. investigated perturbed Rydberg series of doubly excited Sr-I [34].

2. THEORY:

Martin's expression provides the most precise results of spectrum for single valence electron atoms. For multivalence electron systems the results from this formula are not appreciable in most cases. WBEPMT provides a better platform to handle such situations. In WBEPMT there are two kinds of electrons: WBE and NWBEs. The electron which is most likely to be ionized or is excited most easily is called the WBE and the remaining electrons are the NWBEs. The nucleus and NWBEs form the ionic core; the WBE is under the influence of the ionic core's potential field. It is based on this assumption that every atom behaves approximately as a Hydrogenic atom. The +(Z-1) electrons with the nucleus form the ionic core, while the WBE moves under the influence of effective potential of this ionic core. This effective potential is given by [35]:

$$V(r) = -\frac{Z}{r} + \frac{Y}{r^2} \quad (1)$$

where

$$Y = \frac{m(m+1)+2ml}{2} \quad (2)$$

bound electron (WBE). Many theoretical types of research were conducted based on this concept and results show good agreement between theoretical and earth

In equation (1) the two terms on the right-hand side represent Coulombic potential and polarization potential respectively. The polarization potential is created by the dipole formed between ion-core and WBE. In the first term, r is the separation distance between WBE and the nucleus, Z is the effective nucleus charge and in Y, *l* is the angular quantum number of WBE, and m is a measurable factor needed to be determined, not necessarily an integer.

The energy formula for WBEs is:

$$E = -\frac{1}{2}\left(\frac{Z^*}{n^*}\right)^2 \quad (3)$$

Here, n^* is the effective principal quantum number, the Z^* and n^* are unknown values, and are calculated by the transformation between Eigen-values of quantum defect theory (QDT) and WBEPMT, which gives:

$$\frac{Z^*}{n^*} = \frac{Z_o}{n-\delta_n} \quad (4)$$

Here, Z_o is the atomic kernel net charge number and δ_n is the quantum defect in the principal quantum number 'n'. According to this theory, the energy (T) of the perturbed or unperturbed level is the sum of ionization limit (T_{limit}) and Energy (E) of WBE, and is given by [36]

$$T = T_{limit} + E \quad (5)$$

$$T = T_{limit} - \frac{1}{2}\left(\frac{Z_o}{n-\delta_n}\right)^2 \quad (6)$$

The quantum defect (δ_n) are computed by Ritz's formula for unperturbed series and is given as [37].

$$\delta_n = a_o + a_1(n-\delta_o)^{-2} + a_2(n-\delta_o)^{-4} + a_3(n-\delta_o)^{-6} \quad (7)$$

In equation (7) δ_0 is the lowest Rydberg state quantum defect of the series, coefficients ($a_{i's}$, i=0,1,2,3) are obtained by the method of least-square fitting, using the first few experimental values of the spectrum like energy levels series [38]. Many spectral level series in Sr-I are perturbed by foreign level configuration mixing of same parity; for treating such spectral series Martin's expression is combined with Langer's [39]. Zheng et al. proposed an extended Martin's expression for computing perturbed spectral energy levels series [40]. The proposed equation is given below:

$$\delta_n = \sum_{i=1}^{4} a_i\, m^{-2(i-1)} + \sum_{j=1}^{N} \frac{b_j}{m^{-2}-\varepsilon_j} \quad (8)$$

In which, $m = n - \delta_0$ and N is the number of the foreign perturbing levels.

$$\varepsilon_j = \frac{2(T_{limit}-T_{j.per})}{Z_{net}^2} \quad (9)$$

Here, $T_{j.per}$ is the energy of perturbing levels.

By utilizing regularity of change in quantum defects, the radiative lifetimes of the levels can be found by Rykova's expression [41].

$$\tau = \tau_0 (n*)^{\alpha} \quad (10)$$

where τ_o and α are the parameters of Rykova's expression. A least square method can be used to calculate values of the parameters with the help of known lifetimes of the first few levels [42-46].

3. RESULT AND DISCUSSION:

3.1. Spectral Energy Series.

In this work eleven spectral energy series: ***[Kr]5sns$^1S_0$, [Kr]5sns$^3S_1$, [Kr]5snp$^1P^o_1$, [Kr]5snd$^1D_2$, [Kr]5snd$^3D_{(1,2,3)}$, [Kr]5snf $^1F^o_3$, [Kr]5snf $^3F^o_{(2,3,4)}$*** of Sr-I are studied. Among them following five: ***[Kr]5sns$^1S_0$, [Kr]5snp $^1P^o_1$, [Kr]5snd$^1D_2$, [Kr]5snd$^3D_2$, [Kr]5snf $^1F^o_3$,*** are perturbed spectral energy series and the remaining six ***[Kr]5sns$^3S_1$, [Kr]5snd$^3D_{(1,3)}$, [Kr]5snf$^3F^o_{(3,4,5)}$*** are unperturbed. The regularity of quantum defects is evaluated by Martin's formula for unperturbed series but for the perturbed series, Martin's expression was combined with Langer's expression (see equation 8) to evaluate the spectral energies. The effectiveness of evaluation of spectral energies of perturbed series depends upon the correct identification of perturbing levels. The perturbing levels lie within the vicinity of perturbed series having the same total angular quantum number ($l \pm s$) and parity. The perturbing levels of the perturbed series of Sr-I are listed in Table I with their spectral energies in cm^{-1}.

Table I. The perturbed series and perturbing levels of Sr I.

Spectral Perturbed series	*Foreign perturbing levels*	*J*	*Energy [49] (cm^{-1})*
[Kr]5sns (n ≥ 6)$^1S_0$	*[Kr]5s5p2 ^3P*	0	35193.442
	[Kr]5s5p2 ^1S	0	37160.234
[Kr]5snp (n ≥ 5)$^1P^o_1$	*[Kr]5s4d5p $^3D^o$*	1	36264.151
	[Kr]5s4d5p $^3P^o$	1	37302.731
	[Kr]5s4d5p $^1P^o$	1	41172.054
[Kr]5snd (n ≥ 6)$^1D_2$	*[Kr]5s5p2 ^1D*	2	36960.842
	[Kr]5s9d ^3D	2	43804.89
	[Kr]5s10d ^3D	2	44286.91
	[Kr]5s4d2 ^3P	2	44729.627
	[Kr]5s14d ^3D	2	45171.49
	[Kr]5s15d ^3D	2	45276.65
	[Kr]5s16d ^3D	2	45350.35
	[Kr]5s17d ^3D	2	45420.84
	[Kr]5s18d ^3D	2	45479.88
[Kr]5snd (n ≥ 5)$^3D_2$	*[Kr]5s5p2 ^3P*	2	35674.637
	[Kr]5s4d2 ^3P	2	44729.627
[Kr]5snf (n ≥ 5)$^1F^o_3$	*[Kr]5s4d5p $^3F^o$*	3	33589.709
	[Kr]5s4d5p $^3D^o$	3	36559.492

The quantum defects were found by the method described above for unperturbed and perturbed series. The coefficients ***a_i's, b_j's*** and ***δ_0*** were evaluated for perturbed series and are given in Table II-A to II-D. The energy calculations for perturbed series were carried out by taking energies of perturbing levels into account. The number of perturbing levels is two or three, except for the ***[Kr]5snd (n ≥ 6)$^1D_2$*** series, for which the number of perturbed levels is nine. The spectral energies of all eleven series are computed up to n=50 principal quantum number. For perturbed series, energies are also calculated by Martin's formula, the results are shown in third columns of tables III-VII. The fourth columns show the calculated values of energies by Martin + Langer method. It is clear that for perturbed series the latter method gives better results. These results are in good agreement with reference values. Only results of a few levels of perturbed series are shown in tables III-VII. The detailed results of energies for five perturbed and six unperturbed series are given in the supplementary file. In most of the cases, deviation is less than 0.1 cm^{-1} except for a few levels.

Table II-B. The Calculated coefficients given in equations 7 and 8 for both perturbed and unperturbed series

Spectral energy series	*[Kr]5snd^{1}D$_2$ (6≤ n ≤ 50)*	*[Kr]5snd^{3}D$_1$ (4≤ n ≤ 50)*	*[Kr]5snd^{3}D$_2$ (5≤ n ≤ 50)*
a_1	0.657	1.814	1.862
a_2	-83.958	-0.406	-3.905
a_3	2103.955	5.131	84.410
a_4	-36519.092	-1.720	-540.596
b_1	-0.145		3.02E-04
b_2	-1.82E-06		6.52E-04
b_3	-2.49E-06		
b_4	1.28E-05		
b_5	5.05E-07		
b_6	3.50E-06		
b_7	2.27E-05		
b_8	4.87E-06		
b_9	1.21E-06		
δ_0	1.793	2.012	1.829

Table II-C. The Calculated coefficients are given in equations 7 and 8 for both perturbed and unperturbed series

Spectral energy series	*[Kr]5snd^{3}D$_3$ (5≤ n ≤ 50)*	*[Kr]5snf^{1}F$^o_3$ (4≤ n ≤ 50)*	*[Kr]5snf^{3}F$^o_2$ (4≤ n ≤ 50)*
a_1	1.834	-2.417	0.112
a_2	-1.943	-22.546	-0.370
a_3	31.983	-111.833	1.090
a_4	-134.319	-2569.361	-6.384
b_1		-0.358	
b_2		0.058	
b_3			
b_4			
b_5			
b_6			
b_7			
b_8			
b_9			
δ_0	1.825	-0.143	0.091

Table II-D. The Calculated coefficients are given in equations 7 and 8 for both perturbed and unperturbed series

Spectral energy series	*[Kr]5snf^{3}F$^o_3$ (4≤ n ≤ 50)*	*[Kr]5snf^{3}F$^o_4$ (4≤ n ≤ 50)*
a_1	0.112	0.113
a_2	-0.408	-0.466
a_3	2.074	3.195
a_4	-14.504	-22.176
b_1		
b_2		
b_3		
b_4		
b_5		
b_6		
b_7		
b_8		
b_9		
δ_0	0.091	0.090

Table III. Spectral Energies for *[Kr] 5sns^{1}S$_o$* in cm^{-1} Calculate from Martin and Extended Martin formula (T_{lim}= 45932.2036 cm^{-1})

*n**	Spectral Energies			*ΔT= T_{Exp}-T_{per}*
	T_{Exp} [49]	*T_{unper}*	*T_{per}*	*cm^{-1}*
2.67	30591.825	30591.825	30591.825	0.00
3.83	38444.013	38444.013	38444.013	0.00
4.74	41052.324	41052.324	41052.324	0.00
5.74	42596.572	42596.572	42596.572	0.00
6.73	43512.1658	43527.5198	43512.1658	0.00
7.73	44097.1224	44118.8382	44097.1224	0.00
8.73	44492.8348	44515.4340	44493.1477	-0.31
9.73	44773.6707	44794.0452	44773.5882	0.08
10.73	44979.4540	44997.3146	44979.3948	0.06
11.73	45134.9242	45150.2583	45134.8740	0.05
12.73	45255.2295	45268.2957	45255.1887	0.04
13.73	45350.2296	45361.3462	45350.1942	0.04
14.73	45426.5505	45436.0283	45426.5211	0.03
15.73	45488.7860	45496.8977	45488.7628	0.02
16.73	45540.2024	45547.1755	45540.1830	0.02

Table IV. Spectral Energies for *[Kr] 5snp $^1P^o_1$* in cm^{-1} Calculate from Martin and Extended Martin formula (T_{lim}= 45932.2036 cm^{-1})

*n**	Spectral Energies			$\Delta T = T_{Exp}-T_{per}$
	T_{Exp} [49]	T_{unper}	T_{per}	cm^{-1}
2.13	21698.452	21698.452	21698.452	0.00
3.05	34098.404	34098.404	34098.404	0.00
3.95	38906.858	38906.858	38906.858	0.00
5.62	42462.136	42462.136	42462.136	0.00
6.49	43328.04	43858.20	43328.04	0.00
7.42	43938.201	44509.675	43938.201	0.00
8.37	44366.42	44870.34	44366.42	0.00
9.34	44675.737	45095.730	44674.465	1.27
10.32	44903.5	45249.0	44901.7	1.76
11.30	45075.29	45359.56	45073.48	1.81
12.29	45207.83	45442.97	45206.08	1.75
13.28	45311.99	45507.98	45310.43	1.56
14.28	45395.34	45559.96	45393.95	1.39
15.27	45463.02	45602.39	45461.79	1.23
16.27	45518.64	45637.58	45517.62	1.02
17.27	45565	45667	45564	0.90

Table V. Spectral Energies for *[Kr] 5snd $^1D_2$* in cm^{-1} Calculate from Martin and Extended Martin formula (T_{lim}= 45932.2036 cm^{-1})

*n**	Spectral Energies			$\Delta T = T_{Exp}-T_{per}$
	T_{Exp} [49]	T_{unper}	T_{per}	cm^{-1}
4.21	39733.067	39733.067	39733.067	0.00
5.17	41831.448	41831.448	41831.448	0.00
6.14	43021.058	43021.058	43021.058	0.00
7.10	43755.755	43755.755	43755.755	0.00
8.05	44239.4549	44244.7446	44239.4549	0.00
9.00	44578.689	44587.249	44578.689	0.00
9.98	44829.6648	44836.3202	44829.6648	0.00
10.92	45012.0249	45022.8759	45012.0249	0.00
11.87	45153.2785	45166.0442	45153.2785	0.00
12.81	45263.6196	45278.1933	45263.6196	0.00
13.87	45362.1272	45367.6056	45362.1272	0.00
14.83	45433.2717	45439.9908	45433.2717	0.00
15.80	45492.6101	45499.3841	45492.6101	0.00
16.77	45542.2955	45548.6996	45542.0925	0.20
17.75	45584.1831	45590.0826	45583.9436	0.24

Table VI. Spectral Energies for *[Kr] 5snd $^3D_2$* in cm^{-1} Calculate from Martin and Extended Martin formula (T_{lim}= 45932.2036 cm^{-1})

*n**	Spectral Energies			$\Delta T = T_{Exp}-T_{per}$
	T_{Exp} [49]	T_{unper}	T_{per}	cm^{-1}
3.17	35021.989	35021.989	35021.989	0.00
4.19	39690.802	39690.802	39690.802	0.00
5.20	41869.27	41869.27	41869.27	0.00
6.19	43070.268	43070.268	43070.268	0.00
7.18	43804.89	43807.02	43804.89	0.00
8.17	44286.91	44292.21	44286.91	0.00

Table VII Spectral Energies for *[Kr] 5snf $^1F^o_3$* in cm^{-1} Calculate Martin and Extended Martin formula (T_{lim}= 45932.2036 cm^{-1})

*n**	Spectral Energies			$\Delta T = T_{Exp}-T_{per}$
	T_{Exp} [49]	T_{unper}	T_{per}	cm^{-1}
4.14	39539.013	39539.013	39539.013	0.00
4.99	41519.04	41519.04	41519.04	0.00
5.96	42839.589	42839.589	42839.589	0.00
6.94	43656.219	43656.219	43656.219	0.00
7.94	44189.889	44188.765	44189.889	0.00
8.93	44556.48	44554.55	44556.48	0.00
9.93	44818.77	44816.50	44818.77	0.00
10.92	45012.82	45010.45	45012.77	0.05
11.92	45160.29	45158.02	45160.23	0.06
12.92	45274.97	45272.89	45274.91	0.06
13.92	45365.9	45364.0	45365.8	0.07
14.92	45439.16	45437.52	45439.14	0.02
15.92	45499.11	45497.66	45499.09	0.02
16.92	45548.76	45547.48	45548.74	0.02
17.92	45590.32	45589.21	45590.32	0.00
18.92	45625.48	45624.51	45625.50	-0.02
19.91	45655.49	45654.64	45655.51	-0.02

3.2. Radiative lifetimes:

To calculate lifetimes of energy levels in each series, equation (10) is used [42-48]. The necessary experimental data of first few energy levels was obtained from NIST. The coefficients of Rykova's formula (10) were determined by least-square fitting and are given in Table VIII.

Table VIII. The coefficients for the calculation of Radiative lifetimes of Rydberg series.

Spectral Energy Series	τ_o	α
[Kr]5sns $^3S_1$ *($19 \le n \le 50$)*	1.630E-08	1.718
[Kr]5snp $^1P^o_1$ *($5 \le n \le 50$)*	1.585E-11	4.234
[Kr]5snd $^1D_2$ *($13 \le n \le 50$)*	2.250E-10	2.862
[Kr]5snd $^3D_2$ *($5 \le n \le 50$)*	3.614E-10	3.207
[Kr]5snd $^3D_3$ *($18 \le n \le 50$)*	2.065E-12	4.313
[Kr]5snf $^1F^o$ $_{(3,4,5)}$ *($4 \le n \le 50$)*	5.354E-10	2.805

Once the coefficient τ_o and α are known the lifetimes up to ***n=50*** principal quantum number of the following series [***[Kr] 5sns$^3S_1$, [Kr] 5snp$^1P^o_1$, [Kr] 5snd$^1D_2$, [Kr] 5snd$^3D_{2,3}$, [Kr] 5snf $^1F^o_{3,4,5}$)***, were determined for Sr-I spectral energy series. The calculated lifetimes are listed in Tables IX-XI.

Table IX. Radiative lifetimes for: *[Kr] 5sns $^3S_1$ and [Kr] 5snd $^1D_2$* in Sr-I

[Kr] 5sns $^3S_1$			***[Kr] 5snd $^1D_2$,***		
n*	τ_{nS} [47]	$\tau_{cal(nS)}$	***n****	τ_{nS} [42-44]	$\tau_{cal(nS)}$
19	1830	1833.63	13	210	214.27
20	2040	2039.83	14	308	273.8
21	2240	2255.12	15	340	343.39
22	2480	2479.35	16	35	423.79
23	2650	2712.4	17	517	515.73
24		2954.12	18	640	619.92
25		3204.4	19	738	737.09
30		4580.43	20	866	867.92
40		7920.92	21	1023	1013.14
35		6155.76	22	1190	1173.42
40		7920.92	25		1751.52
45		9868.17	30		3079.64
50		11991	35		3753.99
			40		5843.08
			45		8572.03
			50		12019.2

Table X. Radiative lifetimes for: *[Kr] 5snp $^1Po_1$* and *[Kr] 5snd $^3D_2$* in Sr-I

[Kr] 5snp $^1Po_1$			***[Kr] 5snd $^3D_2$***		
n*	τ_{nS} [48]	$\tau_{cal(nS)}$	***n****	τ_{nS} [47]	$\tau_{cal(nS)}$
6	27.1	23.73	5	16	14.65
7	42.7	43.6	6	32	35.87
8	81	76.74	7	74	71.4
9	125	127.78	8	108	125.25
10	170	202.76	9	232	201.87
11	250	308.67	10		305.9
12	510	453.53	15		1406.9
13	690	646.35	20		3947.04
14	925	897.21	25		8608.38
15	1190	1217.26	30		16113.8
16	1550	1618.73	35		27219.5
17	1890	2114.98	40		42710.6
18	2730	2720.48	45		63397.1
19	4270	3450.88	50		90111.5
20		4322.95			
21	5650	5354.68			
22		6565.22			
23	7500	7974.94			
25		11479.5			
30		18810.5			
35		38398.3			
40		70717.6			
45		120589			
50		193702			

Table XI. Radiative lifetimes for: *5snd $^3D_3$* and *[Kr] 5snf $^1F^o_{(3,4,5)}$* in Sr-I

[Kr] 5snd $^3D_3$			***[Kr] 5snf $^1F^o_{(3,4,5)}$***		
n*	τ_{nS} [47]	$\tau_{cal(nS)}$	***n****	τ_{nS} [48]	$\tau_{cal(nS)}$
18	63	336.68	4	31.3	28.87
19	65	436.14	5	45	48.56
20	48	556.78	6	78	79.97
21	770	701.57	7	120	122.94
22	940	873.7	8	179	178.83
23	1130	1076.58	9	255	249.09
24	1330	1313.88	10	350	335.15
25	1580	1589.45	11	430	438.41
26	1770	1907.44	15		1050.82
30		3694.35	20		2363.2
35		7479.22	25		4429.21
40		13711.9	30		7398.13
45		23328.9	35		11413.4
50		37443.05	40		16613.9
			45		23134.7
			50		31107.3

4. CONCLUSION:

In this theoretical spectral analysis of quantum defects of neutral Strontium (Sr-I), WBEPMT was implemented for both perturbed and unperturbed series ***[Kr] 5snl*** *(l=s, p, d, f)* ***($^1S_0$,$^3S_1$,$^1P^o_1$,$^1D_2$,$^3D_{(1,2,3)}$,$^1F^o_3$,$^3F^o_{(2,3,4)}$).*** Martin's expression works well for the unperturbed series ***[Kr]5sns$^3S_1$, [Kr]5snd$^3D_{(1,3)}$, [Kr]5snf$^3F^o_{(2,3,4)}$,*** however for the perturbed

series $[Kr]5sns\,^1S_0$, $[Kr]5snp\,^1P^o_1$, $[Kr]5snd\,^1D_2$, $[Kr]5snd\,^3D_2$, $[Kr]5snf\,^1F^o_3$ an extended version of Martin's expression is used. Rydberg spectral energy for the above-mentioned series were determined; the deviations in calculated values were less than 0.1 cm^{-1} in most cases.

The radiative lifetimes of eight of the series mentioned above are also calculated up to n = 50. Due to perturbations, the radiative lifetimes deviate significantly from simple scaling law (n^{*3}). Despite this, our results show good agreement with experimental values under the limit of uncertainties incurred by foreign level perturbers. The lifetimes are more sensitive than the spectral energies to the influence of foreign level perturbations.

To conclude, in spectral energy series of Sr-I strong perturbations occur due to foreign levels which have the same total angular momentum and parity and lie in the vicinity of the series. This configuration mixing in the spectral series not only affects the spectral energies and wave functions but also affects the radiative lifetimes.

REFERENCES:

1. Bublitz GU, Boxer SG., *"Stark spectroscopy: applications in chemistry, biology, and materials science". Annual review of physical chemistry.* **48(1):**213-42, (1997).
2. Fabre C, Haroche S, Goy P., *"Millimetre spectroscopy in sodium Rydberg states: Quantum-defect, fine-structure, and polarizability measurements". Physical Review A.* **18(1):**229, (1978).
3. Hänsch TW, Lee SA, Wallenstein R, Wieman C. *"Doppler-Free Two-Photon Spectroscopy of Hydrogen 1 S− 2 S". Physical Review Letters.* **34(6):**307, **(**1975**).**
4. Larsson J. *"Optical Double Resonance and Level-Crossing Spectroscopy Using Pulsed Laser Excitation". Lund Reports in Atomic Physics.* 1987**.**
5. Grant IP, McKenzie BJ, Norrington PH, Mayers DF, Pyper NC. *"An atomic multiconfigurational Dirac-Fock package". Computer Physics Communications.*207-31, **(**1980**).**
6. Máššik J, Hubačc I, Mach P. *"Applicability of quasi-degenerate many- body perturbation theory to quasi-degenerate electronic states: The H4 model revisited". International Journal of Quantum Chemistry.* **53(2):**207-28, **(**1995**).**
7. Fischer CF, Hansen JE. *"Theoretical oscillator strengths for the resonance transitions in the Zn i isoelectronic sequence". Physical Review A.* **17(6):**1956, **(**1978**).**
8. Hinze J, Roothaan CC. *"Multi-configuration self-consistent-field theory". Progress of Theoretical Physics Supplement.* **40**:37-5, **(**1967**).**
9. Bartlett RJ, Purvis GD. *"Many- body perturbation theory, coupled- pair many- electron theory, and the importance of quadruple excitations for the correlation problem". International Journal of Quantum Chemistry.* **14(5):**561-81, **(**1978**).**
10. Seaton MJ. *"Quantum defect theory". Reports on Progress in Physics.* **46(2)**:167, **(**1983**).**
11. Zheng NW, Wang T, Ma DX, Zhou T, Fan J. *"Weakest bound electron potential model theory". International journal of quantum chemistry.* **98(3):**281-90, **(**2004**).**
12. Zheng N, Fan J, Ma D, Wang T. *"Theoretical study of energy levels and transition probabilities of singly ionized aluminium (Al II)". Journal of the Physical Society of Japan.* **72(12):**3091-6, **(**2003**).**
13. Zheng N, Sun Y. *"The regularities of the Rydberg energy level of many-valence electron atom Al". Science in China Series B: Chemistry.* **43(2):**113-20, **(**2000**).**
14. Tian-Yi Z, Neng-Wu Z. *"Theoretical study of energy levels and transition probabilities of boron atom". Acta Physica Polonica-Series A General Physics.* **116(2):**141, **(**2009**).**
15. Zheng NW, Li Z, Ma D, Zhou T, Fan J. *"Theoretical study of energy levels of atomic Ga". Canadian journal of physics.* **82(7):**523-9, **(**2004**).**
16. Yildiz M. *"Energy Levels and Atomic Lifetimes of Rydberg States in Neutral Indium". Acta Physica Polonica A.***123(1):**25-30, **(**2013**).**
17. Zhou C, Liang L, Zhang L. *"Theoretical calculation of energy levels and radiative lifetimes of Tl I". Chinese Optics Letters.* **6(3):**161-4, **(**2008**).**
18. Raza. S, Ali. N, Hameed. M N. *"Spectral Energies and Radiative Lifetimes of Rydberg States in Neutral Hydrogen". The Journal of Natural Sciences Research*. **8, (No 14)**: 37-45, (2018).
19. Ali. N, Hameed. M N, Raza. S, *"Theoretical Investigation of Radiative Lifetimes and Rydberg Levels Sequence in Indium I". The Journal of Natural Sciences Research.* **Volume 8, (No 17):** 29-50, (2018).
20. Zheng N, Ma D, Yang R, Zhou T, Wang T, Han S. *"An efficient calculation of the energy levels of the carbon group". The Journal of Chemical Physics*. **113(5):**1681-7, (2000).
21. Zhang T, Zheng N, Ma D. *"Theoretical calculation of energy levels of Sr I". Physica Scripta*. **75(6):**763, (2007).
22. ZHOU C, CAO JJ, LIANG L, ZHANG L. *"Theoretical calculation of energy levels of Pb III". Turkish Journal of Physics*. **35(1):**37-42, (2011).
23. Zheng NW, Zhou T, Yang R, Wang T, Ma D. *"Analysis of the bound odd-parity spectrum of krypton by weakest bound electron potential model theory". Chemical Physics*. **258(1):**37-46, (2000).
24. Poirier M, Semaoune R. *"Analysis of electronic correlations in large-angular-momentum states using the Coulomb Green's function: positions of low doubly excited states in alkaline-earth atoms". Journal of Physics B: Atomic, Molecular and Optical Physics*. **31(7):**1443, (1998).
25. Poirier M, Semaoune R. *"Polarization effects in autoionization processes of alkaline-earth-metal atoms: Coulomb Green's-function analysis*". *Physical Review A*. **59(5):**3471, (1999).
26. Cooke WE, Gallagher TF, Edelstein SA, Hill RM. *"Doubly excited autoionizing Rydberg states of Sr". Physical Review Letters.* **40(3):**178, (1978).
27. Bi-ru W, Zhang-jian C, Yun-Fei X, Jun-feng S. *"An experimental investigation of the 6sns and 6snd doubly excited autoionizing states of strontium". Journal of Quantitative Spectroscopy and Radiative Transfer.* **49(3):**237-9, (1993).
28. Griesmann U, Esser B, Hormes J. *"The total photo absorption cross section of strontium atoms in the range of the 5pns doubly excited states". Journal of Physics B: Atomic, Molecular and Optical Physics*. **27(17)**:3939, (1994).

29. Aymar M, Greene CH, Luc-Koenig E. "*Multichannel Rydberg spectroscopy of complex atoms"*. *Reviews of Modern Physics*. **68(4):**1015, (1996).
30. Kompitsas M, Cohen S, Nicolaides CA, Robaux O, Aymar M, Camus P. *"Observation and theoretical analysis of the odd J= 3 autoionizing spectrum of Sr up to the 4d threshold". Journal of Physics B: Atomic, Molecular and Optical Physics.* 23(14):2247, (1990).
31. Goutis S, Aymar M, Kompitsas M, Camus P. *"The perturbed even-parity J= 1e, 2e autoionizing spectra of strontium below the 4d5/2 threshold: observation and theoretical analysis". Journal of Physics B: Atomic, Molecular and Optical Physics*. **25(16):**3433, (1992).
32. Kompitsas M, Goutis S, Aymar M, Camus P. *"The even-parity J= 0 autoionizing spectrum of strontium below the 4d5/2 threshold: observation and theoretical analysis". Journal of Physics B: Atomic, Molecular and Optical Physics.* **24(7):**1557, (1991).
33. Cohen S, Aymar M, Bolovinos A, Kompitsas M, Luc-Koenig E, Mereu H, Tsekeris P. *"Experimental and theoretical analysis of the 5pnpJ= 0 e, 1 e, 2 e autoionizing spectrum of Sr". The European Physical Journal D-Atomic, Molecular, Optical and Plasma Physics.* **13(2):**165-80, (2001).
34. Zhang T, Zheng N, Ma D. *"Theoretical calculation of energy levels of Sr I". Physica Scripta*. **75(6):**763, (2007).
35. Zheng NW, Wang T, Ma DX, Zhou T, Fan J. *"Weakest bound electron potential model theory". International journal of quantum chemistry*. **98(3)**:281-90, (2004).
36. Zheng N, Ma D, Yang R, Zhou T, Wang T, Han S. *"An efficient calculation of the energy levels of the carbon group". The Journal of Chemical Physics.* **113(5):**1681-7, (2000).
37. Martin WC. *"Series formulas for the spectrum of atomic sodium (Na I)". JOSA*. **70(7)**:7848, (1980).
38. Shizhong H, Qiufeng S. *"Calculation of the Rydberg Energy Levels for Francium Atom". Physics Research International.* (2010).
39. Langer R M. *"A generalization of the Rydberg formula". Phys. Rev*. **35,** 649-768, (1930)
40. Zheng N, Ma D, Yang R, Zhou T, Wang T, Han S. *"An efficient calculation of the energy levels of the carbon group". The Journal of Chemical Physics.* **113(5):**1681-7, (2000).
41. Verolaĭnen YF, Nikolaich AY. *"Radiative lifetimes of excited states of atoms". Soviet Physics Uspekhi.* **25(6):**431, (1982).
42. Photochemistry M. A Brief Review of Multiphoton Absorption Theory 2.1 Coherent processes 2.2 Incoherent processes. Advances in Multi-photon Processes and Spectroscopy. **1:239**, (1984).
43. Grafström P, Zhan-Kui J, Jönsson G, Levinson C, Lundberg H, Svanberg S. *"Natural radiative lifetimes in the interacting 5 s n d 2 1, 3 sequences in Sr". Physical Review* A. **27(2):**947, (1983).
44. Osherovich AL, Verolainen YF, Pulkin SA, Privalov VI, Lupekhin SM. *"Investigation of the radiative lifetimes of the excited states of Sr i and Sr ii". Optics and Spectroscopy*. **46**:134-6, (1979).
45. Kelly FM, Koh TK, Mathur MS. *"Lifetimes and oscillator strengths in singlet levels of strontium". Canadian Journal of Physics.* **52(9):**795-802, (1974).
46. Vaeck N, Godefroid M. *"Multiconfiguration Hartree-Fock calculations for singlet terms in neutral strontium". Physical Review A.* **38(6)**:2830, (1988).
47. Kunze S, Hohmann R, Kluge HJ, Lantzsch J, Monz L, Stenner J, Stratmann K, Wendt K, Zimmer K. *"Lifetime measurements of highly excited Rydberg states of strontium I". Zeitschrift für Physik D Atoms, Molecules and Clusters.* **27(2):**111-4, (1993).
48. Jönsson G, Levinson C, Persson A, *"Wahlström CG. Natural radiative lifetimes in the* $^1P_1$ *and* $^1F_3$ *sequences of Sr I". Zeitschrift für Physik A Atoms and Nuclei*. **316(3):**255-8, (1984).
49. Kramida, A., Ralchenko, Yu., Reader, J., and NIST ASD Team *NIST Atomic Spectra Database (ver. 5.6.1), [Online]. Available: https://physics.nist.gov/asd. National Institute of Standards and Technology, Gaithersburg, MD*. DOI: https://doi.org/10.18434/T4W30F (2018).