SOME COMMON RANDOM FIXED POINTS THEOREMS OF QUASI-CONTRACTION RANDOM OPERATORS IN METRIC SPACE AND RANDOM WELL-POSED

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ABSTRACT: *Our aim in this paper is to prove the random coincidence points for two random operators under quasi contraction conditions in metric space. The random well-posed fixed-point problem is best studied by proving applications that are related to common random fixed-point results.*

Keywords: Random well-posed; Common random fixed-points.

1. INTRODUCTION AND PRELIMINARIES:

In the 1950s, Špaček [1] and Hanš [2, 3] reported the first work on random fixed-point (RFP) theorems at the Prague School of Probabilities. Following the article published by Bharucha-Reid [4] in 1976, the interest in these problems grew tremendously. For instance, Chauhan [5] focused on the common fixed-point (CFP) theorem for 4 continuous random operators that satisfies some contractive criteria in Separable Hilbert space. In 2014, Ahmed [6] proved the existence of CFP for random mappings satisfying new type of rational contractive conditions in S-metric space.

In 2016, Abed and Ajeel [7] proved RFP theorem for Banach operator which is defined on separable closed subset of a complete p-normed space.

Rashwan & Hammed [8], in 2017, demonstrated a unique common RFP theorem for 4 loosely compatible mappings in cone random metric spaces based on an implicit relation. Abed et. at [9] focused on two continuous random operators to prove the common RFP theorem in complete p-normed space under quasi contraction condition.

This article focused on common RFP generation for two random operators under quasi contraction condition in metric space. Also studied was the well-posedness problem of RFPs.

In this article, X will be the metric space, $\emptyset \neq A \subseteq X$ be a closed, (Ω,Σ) will be the measurable space with Σ which is a sigma algebra of subsets of Ω. 2^X represents the classes of all X subsets, while CB(X) represents the classes of the whole bounded non-empty closed X subsets. $RF(S,T)$ stands for the common RFPs of S & T and $RC(S,T)$ is the set of random coincidence points of S & T.

We need the following definitions and facts:

***Definition (1.1)*: [10]**

"A mapping $F\colon \Omega \to 2^X$ is called measurable (respectively, weakly measurable) if, for any closed (respectively, open) subset B of, $F^{-1}(B) = \{\omega \in \Omega : F(\omega) \cap B \neq \emptyset\} \in \Sigma$."

***Definition (1.2)*: [11]**

"A mapping $\delta\colon \Omega \to X$ is called a measurable selector of a measurable mapping $F\colon\Omega \to 2^X$ if δ measurable and $\delta(\omega) \in F(\omega)$ for each $\omega \in \Omega$."

***Definition (1.3)*: [12]**

"A mapping $h\colon \Omega \times X \to X$(or $G\colon \Omega \times X \to CB(X)$) is called a random operator if for any $x \in X$, $h(.,x)$(respectively G(.,x) is measurable ."

Definition (1.4): [13]

"A measurable mapping $\delta\colon \Omega \to A$ is called random fixed point of a random operator $h\colon \Omega \times X \to X$ (or $G\colon \Omega \times X \to CB(X)$ if for every $\omega \in \Omega$, $\delta(\omega) = h(\omega, \delta(\omega))$ (respectively$\delta(\omega) \in G(\omega, \delta(\omega))$."

Definition (1.5): **[14]**

"A measurable mapping $\delta\colon \Omega \to A$ is called random coincidence point of a random operator $h\colon \Omega \times A \to A$ and $G\colon \Omega \times A \to A$ if for every $\omega \in \Omega$, $h(\omega, \delta(\omega)) = G(\omega, \delta(\omega))$."

Definition (1.6): **[14]**

"A measurable mapping $\delta\colon \Omega \to A$ is called common random fixed point of a random operator $h\colon \Omega \times A \to X$ and $G\colon \Omega \times A \to A$ if for every $\omega \in \Omega$
$\delta(\omega) = h(\omega, \delta(\omega)) = G(\omega, \delta(\omega))$."

Now, a new type of random operators will be defined.

***Definition (1.7)*:**

Let$(X,\, d)$ be a metric space. Let $T, I\colon \Omega \times A \longrightarrow A$ be two random operators. The random operator T is called I-quasi Contraction operator if we have:

$$d(T(\gamma,x),T(\gamma,y)) \leq k\, max \begin{Bmatrix} d(I(\gamma,x),I(\gamma,y)), d(I(\gamma,x),T(\gamma,x)), d(I(\gamma,y),T(\gamma,y)) \\ , d(I(\gamma,x),T(\gamma,y)), d(I(\gamma,y),T(\gamma,x)), \end{Bmatrix} \quad (1.1)$$

Where, $0 \leq k < \frac{1}{2}$ For all $x, y \in A$.

Definition (1.8):

"Let A be a nonempty subset of a metric space X and let S and T be self-mappings of A the pair (S,T) is said to be:

1) Weakly compatible [15] if they commute at their coincidence points, i.e., $STx = TSx$ for all x satisfying$S(x) = T(x)$.

2) R-weakly commuting maps [16] if for all $x \in A$ there exists $R > 0$ such that $d(STx, TSx) < Rd(Sx, Tx)$, if R =1, then the maps are called weakly commuting."

These definitions was as captured by [15, 17], respectively:

Definition (1.9):

"A random operators $h, G\colon \Omega \times X \longrightarrow X$ are said to be R-weakly commute (or Weakly Compatible) if $h(\omega,.)$ and $G(\omega,.)$ are R-weakly commute (respectively weakly compatible) for each $\in \Omega$."

2. RANDOM COINCIDENCE THEOREMS

We prove that:

***Theorem (2.1)*:**

Let $\emptyset \neq A \subseteq X$ for fixed $\gamma \in \Omega$, the mappings

$T, I(\gamma,.): A \longrightarrow A$ satisfy the condition (1.1) . If $cl(T(\gamma,A)) \subseteq I(\gamma,A)$ and $cl(T(\gamma,A))$ is separable complete subspace of A. Then $RC(T,I) \neq \emptyset$.

Proof:

Let $\delta_\circ: \Omega \longrightarrow A$ be arbitrary measurable mapping.

Then, a sequence of measurable maps $\delta_n: \Omega \longrightarrow A$ was constructed.

Since $cl(T(\gamma,A)) \subseteq I(\gamma,A)$, then we can find $\delta_1: \Omega \longrightarrow A$ such that

$T(\gamma, \delta_\circ(\gamma)) = I(\gamma, \delta_1(\gamma))$.

A sequence of measurable mappings $\delta_n: \Omega \longrightarrow A$ was constructed such that

$$T(\gamma, \delta_{2n-1}(\gamma)) = I(\gamma, \delta_{2n}(\gamma)) \quad (2.1)$$

Hence, the sequence of functions for $\gamma \in \Omega$, $\{y_n(\gamma)\}$ can be defined such that

$$y_{2n}(\gamma) = T(\gamma, \delta_{2n}(\gamma)) = I(\gamma, \delta_{2n+1}(\gamma)) \quad (2.2)$$

$$d(y_{2n}(\gamma), y_{2n+1}(\gamma)) = d\left(T(\gamma,\delta_{2n}(\gamma)), T(\gamma,\delta_{2n+1}(\gamma))\right)$$
$$\leq k \max\{ d\left(I(\gamma,\delta_{2n}(\gamma)), I(\gamma,\delta_{2n+1}(\gamma))\right), d\left(I(\gamma,\delta_{2n}(\gamma)), T(\gamma,\delta_{2n}(\gamma))\right),$$
$$d\left(I(\gamma,\delta_{2n+1}(\gamma)), T(\gamma,\delta_{2n+1}(\gamma))\right), d\left(I(\gamma,\delta_{2n}(\gamma)), T(\gamma,\delta_{2n+1}(\gamma))\right),$$
$$d\left(I(\gamma,\delta_{2n+1}(\gamma)), T(\gamma,\delta_{2n}(\gamma))\right)\}$$
$$= k \max\{ d\left(T(\gamma,\delta_{2n-1}(\gamma)), T(\gamma,\delta_{2n}(\gamma))\right), d\left(T(\gamma,\delta_{2n-1}(\gamma)), T(\gamma,\delta_{2n}(\gamma))\right),$$
$$d\left(T(\gamma,\delta_{2n}(\gamma)), T(\gamma,\delta_{2n+1}(\gamma))\right), d\left(T(\gamma,\delta_{2n-1}(\gamma)), T(\gamma,\delta_{2n+1}(\gamma))\right),$$
$$d\left(T(\gamma,\delta_{2n}(\gamma)), T(\gamma,\delta_{2n}(\gamma))\right)\}$$

Using triangle inequality, we get

$$\leq k \max\{ d\left(T(\gamma,\delta_{2n-1}(\gamma)), T(\gamma,\delta_{2n}(\gamma))\right), d\left(T(\gamma,\delta_{2n}(\gamma)), T(\gamma,\delta_{2n+1}(\gamma))\right),$$
$$d\left(T(\gamma,\delta_{2n-1}(\gamma)), T(\gamma,\delta_{2n}(\gamma))\right) + d\left(T(\gamma,\delta_{2n}(\gamma)), T(\gamma,\delta_{2n+1}(\gamma))\right)\}$$
$$= k[d\left(T(\gamma,\delta_{2n-1}(\gamma)), T(\gamma,\delta_{2n}(\gamma))\right) + d\left(T(\gamma,\delta_{2n}(\gamma)), T(\gamma,\delta_{2n+1}(\gamma))\right)]$$
$$= k[d(y_{2n-1}(\gamma), y_{2n}(\gamma)) + d(y_{2n}(\gamma), y_{2n+1}(\gamma))]$$

Hence, $d(y_{2n}(\gamma), y_{2n+1}(\gamma)) \leq \lambda d(y_{2n}(\gamma), y_{2n-1}(\gamma))$

Where$\lambda = (k/1-k) < 1$.

In general

$d(y_n(\gamma), y_{n+1}(\gamma)) \leq \lambda d(y_n(\gamma), y_{n-1}(\gamma))$

Therefore,

$$d(y_n(\gamma), y_{n+1}(\gamma)) \leq \lambda d(y_n(\gamma), y_{n-1}(\gamma))$$
$$\leq \lambda^2 d(y_{n-1}(\gamma), y_{n-2}(\gamma))$$

$d(y_{n+1}(\gamma), y_n(\gamma)) \leq \lambda^n d(y_0(\gamma), y_1(\gamma))$ for all $\gamma \in \Omega$.

Now, it is time to prove that for $\in \Omega$, $\{y_n(\gamma)\}$ is a Cauchy sequence. For each positive integer p, for $\gamma \in \Omega$

$$d\left(y_n(\gamma), y_{n+p}(\gamma)\right) \leq d(y_n(\gamma), y_{n+1}(\gamma)) + d(y_{n+1}(\gamma), y_{n+2}(\gamma)) + \ldots$$
$$+ d\left(y_{n+p-1}(\gamma), y_{n+p}(\gamma)\right)$$
$$\leq (\lambda^n + \lambda^{n+1} + \ldots + \lambda^{n+p-1})\, d(y_0(\gamma), y_1(\gamma))$$
$$= \lambda^n (1 + \lambda^1 + \ldots + \lambda^{p-1})\, d(y_0(\gamma), y_1(\gamma))$$
$$\leq (\lambda^n/1-\lambda) d(y_0(\gamma), y_1(\gamma)) \text{ for all } \gamma \in \Omega .$$

This implies

$d\left(y_n(\gamma), y_{n+p}(\gamma)\right) \to 0$ as $n \to \infty$ for $\gamma \in \Omega$. (2.3)

It also means that for $\in \Omega$ $\{y_n(\gamma)\}$, is a Cauchy sequence in $T(\gamma, A)$.

Since $cl(T(\gamma,A))$ is a complete subspace of A, the sequence $\{y_n\}$ has a limit $t: \Omega \longrightarrow A$ there exists $t(\gamma) \in cl(T(\gamma,A))$ such that $y_n(\gamma) \to t(\gamma)$ as $n \to \infty$.

Obtained a mapping $u: \Omega \longrightarrow A$ such that $I(\gamma, u(\gamma)) = t(\gamma)$.

Thus we have

$$t(\gamma) = lim_{n\to\infty} y_{2n} = lim_{n\to\infty} T(\gamma, \delta_{2n}(\gamma)) = lim_{n\to\infty} I(\gamma, \delta_{2n+1}(\gamma))$$

Using (2.2) and (1.1), we have

$$d\left(y_{2n}(\gamma), T(\gamma,u(\gamma))\right) = d\left(T(\gamma,\delta_{2n}(\gamma)), T(\gamma,u(\gamma))\right)$$
$$\leq k \max\{d\left(I(\gamma,\delta_{2n}(\gamma)), I(\gamma,u(\gamma))\right), d\left(I(\gamma,\delta_{2n}(\gamma)), T(\gamma,\delta_{2n}(\gamma))\right),$$
$$d\left(I(\gamma,u(\gamma)), T(\gamma,u(\gamma))\right), d\left(I(\gamma,\delta_{2n}(\gamma)), T(\gamma,u(\gamma))\right),$$
$$d\left(I(\gamma,u(\gamma)), T(\gamma,\delta_{2n}(\gamma))\right)\}$$

taking limit as $n \longrightarrow \infty$, we get

$$d\left(t(\gamma), T(\gamma,u(\gamma))\right) \leq k \max\{d\left(t(\gamma), I(\gamma,u(\gamma))\right), d(t(\gamma), t(\gamma)),$$
$$d\left(I(\gamma,u(\gamma)), T(\gamma,u(\gamma))\right), d\left(t(\gamma), T(\gamma,u(\gamma))\right), d\left(I(\gamma,u(\gamma)), t(\gamma)\right)\}$$

From $t(\gamma) = I(\gamma, u(\gamma))$, we have

$d\left(t(\gamma), T(\gamma,u(\gamma))\right) \leq k\, d\left(t(\gamma), T(\gamma,u(\gamma))\right)$

This implies, $(1-k)d\left(t(\gamma), T(\gamma,u(\gamma))\right) \leq 0$

Hence $d\left(t(\gamma), T(\gamma,u(\gamma))\right) = 0 \Rightarrow t(\gamma) = T(\gamma,u(\gamma)) = I(\gamma,u(\gamma))$ (2.4)

Therefore $RC(T,I) \neq \emptyset$.

***Theorem (2.2)*:**

Let $X, A, T, I, cl(T(\gamma,A))$ as in theorem (2.1) .If the pair $\{T,I\}$ is R-weakly commuting (or weakly compatible), then $RF(T) \cap RF(I)$ is a unique singlton element.

Proof:

Theorem 2.1 proves that existence of a random coincidence point $u: \Omega \to A$ of T and I such that $T(\gamma, u(\gamma)) = I(\gamma, u(\gamma))$ for all $\gamma \in \Omega$.

If the pair $\{T, I\}$ is weakly compatible, then

$T(\gamma, I(\gamma, u(\gamma)) = I(\gamma, T(\gamma, u(\gamma))$ from (2.4), we have

$$T(\gamma, t(\gamma)) = I(\gamma, t(\gamma)) \quad (2.5)$$

From (2.4) , (1.1) and (2.5), we have

$$d(t(\gamma), T(\gamma, t(\gamma))) = d\left(T(\gamma,u(\gamma)), T(\gamma,t(\gamma))\right) \leq$$
$$k\, max\{ d\left(I(\gamma,u(\gamma)), I(\gamma,t(\gamma))\right),$$
$$d\left(I(\gamma,u(\gamma)), T(\gamma,u(\gamma))\right), d\left(I(\gamma,t(\gamma)), T(\gamma,t(\gamma))\right),$$
$$d\left(I(\gamma,u(\gamma)), T(\gamma,t(\gamma))\right), d\left(I(\gamma,t(\gamma)), T(\gamma,u(\gamma))\right)\}$$
$$= k \max\{d\left(I(\gamma,u(\gamma)), I(\gamma,t(\gamma))\right),$$
$$d\left(I(\gamma,u(\gamma)), T(\gamma,t(\gamma))\right), d\left(I(\gamma,t(\gamma)), T(\gamma,u(\gamma))\right)\}$$
$$= k \max\{d\left(I(\gamma,u(\gamma)), T(\gamma,t(\gamma))\right),$$
$$d\left(I(\gamma,u(\gamma)), T(\gamma,t(\gamma))\right), d\left(T(\gamma,t(\gamma)), t(\gamma)\right)\}$$
$$= k$$
$$\max\{d\left(t(\gamma), T(\gamma,t(\gamma))\right), d\left(t(\gamma), T(\gamma,t(\gamma))\right), d\left(T(\gamma,t(\gamma)), t(\gamma)\right)\}$$

Then, $(1-k)d\left(t(\gamma), T(\gamma,t(\gamma))\right) \leq 0 \Rightarrow t(\gamma) = T(\gamma,t(\gamma))$

From (2.5) we have

$$t(\gamma) = T(\gamma,t(\gamma)) = I(\gamma,t(\gamma)) \quad (2.6)$$

Thus, $t(\gamma)$ is a common RFP of $T$and I.

Uniqueness:

Let $z(\gamma)$ be another common RFP of $T$and I ,then by using (1.1), we have

$$d(t(\gamma), z(\gamma)) = d\left(T(\gamma,t(\gamma)), T(\gamma,z(\gamma))\right) \leq$$
$$k\, max\{ d\left(I(\gamma,t(\gamma)), I(\gamma,z(\gamma))\right),$$
$$d\left(I(\gamma,t(\gamma)), T(\gamma,t(\gamma))\right), d\left(I(\gamma,z(\gamma)), T(\gamma,z(\gamma))\right),$$
$$d\left(I(\gamma,t(\gamma)), T(\gamma,z(\gamma))\right), d\left(I(\gamma,z(\gamma)), T(\gamma,t(\gamma))\right)\}$$
$$= k \max\{d(t(\gamma), z(\gamma)), d(t(\gamma), t(\gamma)), d(z(\gamma), z(\gamma)),$$
$$d(t(\gamma), z(\gamma)), d(z(\gamma), t(\gamma))\}$$

This implies $(1-k)d(y(\gamma), z(\gamma)) \leq 0 \Rightarrow y(\gamma) = z(\gamma)$.

Assume that $\{T, I\}$ is R-weakly commuting and $u(\gamma)$ is a random coincidence point of T and I, it follows that

$d(T(\gamma, I(\gamma, u(\gamma)), I(\gamma, T(\gamma, u(\gamma))) \le$
$Rd(T(\gamma, u(\gamma), I(\gamma, u(\gamma)) = 0$, thus
$T(\gamma, I(\gamma, u(\gamma)) = I(\gamma, T(\gamma, u(\gamma))$,
Hence, the pair $\{T, I\}$ is said to be loosely compatible. Following similar steps as above, it can be shown that y is a unique common fixed point of T and I.■
Consequently, we will arrive at the following:

***Corollary (2.1)*:**
If A, X and T as in theorem (2.1) and for each $\gamma \in \Omega$, $h(\gamma, .): A \to A$ is (qcr) operator, then $RF(T)$ is a unique element.

Proof:

$$d(T(\gamma, x), T(\gamma, y)) \le k \max \begin{Bmatrix} d(x, y), d(x, T(\gamma, x)), d(y, T(\gamma, y)) \\ , d(x, T(\gamma, y)), d(y, T(\gamma, x)), \end{Bmatrix} \quad (2.7)$$

where , $0 \le k < \frac{1}{2}$ For all $x, y \in A$.
Put $I(\gamma, x) = x$ (the identity random mapping) for all $\gamma \in \Omega$ in theorem (2.1), then the corollary (2.1) stems from theorem (2.1). ■

***Corollary (2.2)*:**
Let $X, A, T, I, cl(T(\gamma, A))$ as in theorem (2.1) .If the pair $\{T, I\}$ meets one of the following criteria:

1. $d(T(\gamma, x), T(\gamma, y)) \le$
$k \max\{d(I(\gamma, x), I(\gamma, y)), d(I(\gamma, x), T(\gamma, x)), d(I(\gamma, y), T(\gamma, y))\}$
2. $d(T(\gamma, x), T(\gamma, y)) \le k \max\{d(I(\gamma, x), T(\gamma, x)), d(I(\gamma, y), T(\gamma, y))\}$
3. $d(T(\gamma, x), T(\gamma, y)) \le$
$k \max\{d(I(\gamma, x), I(\gamma, y)), \frac{1}{2}[d(I(\gamma, x), T(\gamma, x)) + d(I(\gamma, y), T(\gamma, y))], d(I(\gamma, x), T(\gamma, y)), d(I(\gamma, y), T(\gamma, x))\}$
4. $d(T(\gamma, x), T(\gamma, y)) \le$
$k \max\{d(I(\gamma, x), I(\gamma, y)), \frac{1}{2}[d(I(\gamma, x), T(\gamma, x)) + d(I(\gamma, y), T(\gamma, y))], \frac{1}{2}[d(I(\gamma, x), T(\gamma, y)) + d(I(\gamma, y), T(\gamma, x))]\}$
For all $x, y \in X$; $0 < k < 1/2$ .Then $RC(I) \cap RC(T)$ is singleton.

***Corollary (2.3)*:**
Let $X, A, T, I, cl(T(\gamma, A))$ as in corollary (2.2) .If the pair $\{T, I\}$ is weakly compatible, then, $RF(T) \cap RF(I)$ is a unique singlton element.

3. RANDOM WELL-POSED PROBLEM

***Definition (3.1)*:**
Assume (X, d) as a metric space while $T : \Omega \times X \longrightarrow X$ is a random operator; then, the RFP problem of T will be considered well- posed if:

i. T has a unique RFP $\delta : \Omega \longrightarrow X$;
ii. For any measurable $\{\delta_n(\omega)\}$ Sequence of mappings in X such that
iii. $\lim_{n\to\infty} d(T(\omega, \delta_n(\omega)), \delta_n(\omega)) = 0$, we have $\lim_{n\to\infty} d(\delta_n(\omega), \delta(\omega)) = 0$.

***Definition (3.2)*:**
Assume (X, d) as a metric space while the set of random operators in X be represented as $\mathcal{T}$. Then, the RFP of $\mathcal{T}$ will be considered well-posed if :

i. $\mathcal{T}$ has a unique RFP $\delta : \Omega \longrightarrow X$;
ii. for any measurable sequence $\{\delta_n(\omega)\}$ of mappings in X such that $\lim_{n\to\infty} d(T(\omega, \delta_n(\omega)), \delta_n(\omega)) = 0$, $\forall\, T \in \mathcal{T}$. we have $\lim_{n\to\infty} d(\delta_n(\omega), \delta(\omega)) = 0$.

***Theorem (3.1)*:**
If A, X, T and I are as in theorem (2.2), then, the common RFP for the random operators $\{T, I\}$ is considered well-posed.

Proof:
Following Theorem (2.2), it is assumed that T and I have a unique common RFP $\delta: \Omega \longrightarrow A$. Assume $\{\delta_n(\gamma)\}$ to be a sequence of measurable mappings in A such that:

$$\lim_{n\to\infty} d\left(T(\gamma, \delta_n(\gamma)), \delta_n(\gamma)\right) = \lim_{n\to\infty} d\left(I(\gamma, \delta_n(\gamma)), \delta_n(\gamma)\right) = 0$$

By the triangle inequality, (2.2) ,(2.5) and (2.6), we have

$$d(\delta(\gamma), \delta_n(\gamma)) \le d\left(T(\gamma, \delta(\gamma)), T(\gamma, \delta_n(\gamma))\right) + d\left(T(\gamma, \delta_n(\gamma)), \delta_n(\gamma)\right)$$
$$\le k \max\{ d\left(I(\gamma, \delta(\gamma)), I(\gamma, \delta_n(\gamma))\right), d\left(I(\gamma, \delta(\gamma)), T(\gamma, \delta(\gamma))\right), d\left(I(\gamma, \delta_n(\gamma)), T(\gamma, \delta_n(\gamma))\right), d\left(I(\gamma, \delta(\gamma)), T(\gamma, \delta_n(\gamma))\right), d\left(I(\gamma, \delta_n(\gamma)), T(\gamma, \delta(\gamma))\right)\} + d\left(T(\gamma, \delta_n(\gamma)), \delta_n(\gamma)\right)$$
$$\le k\left[d\left(I(\gamma, \delta_n(\gamma)), \delta(\gamma)\right) + d\left(\delta(\gamma), T(\gamma, \delta_n(\gamma))\right)\right] + d\left(T(\gamma, \delta_n(\gamma)), \delta_n(\gamma)\right)$$
$$\le k\left[d\left(I(\gamma, \delta_n(\gamma)), \delta_n(\gamma)\right) + d(\delta_n(\gamma), \delta(\gamma)) + d(\delta(\gamma), \delta_n(\gamma)) + d\left(\delta_n(\gamma), T(\gamma, \delta_n(\gamma))\right)\right] + d\left(T(\gamma, \delta_n(\gamma)), \delta_n(\gamma)\right)$$
$$= Kd\left(I(\gamma, \delta_n(\gamma)), \delta_n(\gamma)\right) + 2Kd(\delta_n(\gamma), \delta(\gamma)) + (1 + K)d\left(\delta_n(\gamma), T(\gamma, \delta_n(\gamma))\right)$$
$$(1 - 2K)d(\delta(\omega), \delta_n(\omega)) \le Kd\left(I(\gamma, \delta_n(\gamma)), \delta_n(\gamma)\right) + (1 + K)d\left(\delta_n(\gamma), T(\gamma, \delta_n(\gamma))\right)$$

Thus, we have, $\lim_{n\to\infty} d(\delta(\omega), \delta_n(\omega)) = 0$, meaning that the common RFP for T *and* I is well-posed.■

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