FLUCTUATIONS IN INTERACTIONS OF PREY PREDATOR SYSTEMS

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ABSTRACT: *In this paper, non-classical system and non-autonomous differential equations are used to take account fluctuations in prey predator system for describing prey predator interactions, which make the description more realistic. Fluctuations involve gradual changes that are represented through sinusoid function, while piecewise function is used to describe sudden changes, whereas the sinusoid function is approximated to three scenarios. The theoretical results explain that Kolmogorov conditions change and it is not applicable according to each scenario. Numerical simulations are performed to explain effects of fluctuations on the dynamical behaviors of gradual changes, the numerical simulation show that the changes in dynamical behavior involve changes the dynamical behaviors from stable case to periodic case, and the changes in the dynamical behaviors include changes size of limit cycles.*

Key words: Fluctuations, Prey predator system, Sinusoid function.

1. INTRODUCTION

Studying the prey-predator interactions has been considerable interest as one of the most important problems in applied mathematics and theoretical ecology [1]. Prey predator system interactions is described through non linear differential equations mathematically [2-8], this field have started since the novel work of Lotka and Volterra which contributed in postulating the dynamical theory of population biology [9].

At the first perusal, the mathematical ecology seem simple, but they considered as complicated problems because of the difficulty in determining ecological principles [8, 10]. The problems inherent in the dynamics of the population have attracted the attention of mathematicians and theoretical ecologists since the pioneering work of Lotka and Volterra.

In reality of ecology, there are several factors cause temporal fluctuations which effect on prey predator system interactions such as hunting and climate. Existence some factors that affect on hunting activity as dry seasons, which make capturing of prey easier due to open habitats, and birth seasons, where prey become more available and easier to capture. Climate changes, which may have strong effects on, prey predator system interactions as floods and hurricanes that lead to destroying the system or immigration from that environment [11]. The study of fluctuations in ecological systems is important as it may change the dynamics of ecological system and more importantly its presence in nature [12]. In the literature, several studies have been used "seasonality" to study the fluctuations in prey predator systems [12, 13]. Most of these studies have widely taken attention the search of chaos cases. Recently, Alebraheem and Abu Hassan [14] have used different system with Crowley - Martin functional response to generate complex dynamical behaviors which involve chaos cases.

The impacting factors can be natural or otherwise which make the interaction of species not constant (i.e. temporal variation) and these effects may be sudden or gradual. Hence, in this paper, we assume the fluctuations in interactions prey predator system to make our assumptions more realistic to investigate the fluctuations in interactions of prey predator system and studying how fluctuations affect the dynamical behaviors in prey predator system.

2. The system

We describe the interactions through couple nonlinear differential equations. Non-dimensional system is presented as follows:

$$\frac{dx}{dt} = x(1-x) - \frac{\alpha x\, y}{1+h\alpha x},$$
$$\frac{dy}{dt} = -uy + \frac{e\alpha xy}{1+h\alpha x} - \frac{e_1\alpha}{(1+h\alpha x)}y^2, \qquad (1)$$

Where α, β, u, e, h, and u are nonnegative constants. The biological meaning of these parameters is explained in Abu Hasan and Alebraheem [15].

The initial conditions of system (1) are:

$\{\ (0) = x_0\ ,\ (0) = y_0$, where $0 \le x_0, y_0 \le 1\}$. (2)

3. FLUCTUATIONS

We use the sinusoid function to describe the fluctuations in interactions of prey predator system and how fluctuations affect on the dynamical behaviors in prey predator system. Sinusoidal functions are mostly used to describe the temporal variations in ecological systems because they describe periodic phenomena [16]. They are assumed for simplicity in describing repetitive oscillation phenomena [17]. However, in this paper we use these functions to describe the gradual changes in the environment. In ecology, there are cases where the changes are sudden. To account for this, we shall use piecewise function in the prey predator systems.

3.1 Fluctuations in Mathematical Systems

The systems are represented through systems of non-autonomous differential equations. To simplify the notations where$Q(t)$ present the fluctuations formula whether periodic function which represent continuous changes, it is described by Sinusoid function $S(t)$, or piecewise function $P(t)$ which represent sudden changes. We investigate the fluctuations in two important parts of the system where the fluctuations are investigated with growth rate of prey in the first case, and with the functional and numerical responses in the second case that consider the link between prey equation and predator equation so affect on both species prey and predator.

3.2 Sinusoid function of gradual changes

Sinusoid function is used to describe the gradual changes, where the sinusoid function is:

$S(t) = 1 + \epsilon \sin(\mu t),$ (3)

Where ϵ represents the degree of seasonality or degree of strength seasonal. The parameter μ is the angular frequency of the fluctuations caused by impacts.

$Q(t) = P(t)$ or $Q(t) = S(t)$ (4)

So the systems become as follows:

In case the fluctuations in growth rate of prey, so the system becomes as follows:

$$\frac{dx}{dt} = xS(t)(1-x) - \frac{\alpha x}{1+h\alpha x}y,$$
$$\frac{dy}{dt} = -uy + \frac{e\alpha x}{1+h\alpha x}y - \frac{e\alpha}{(1+h\alpha x)}y^2 \quad (5)$$

In case the fluctuations in functional and numerical responses, the system becomes as:

$$\frac{dx}{dt} = x(1-x) - \frac{\alpha x S(t)}{1+h\alpha x}y,$$
$$\frac{dy}{dt} = -uy + \frac{e\alpha x}{1+h\alpha x}S(t)y - \frac{e\alpha}{(1+h\alpha x)}y^2 \quad (6)$$

3.3 Approximation method

We use approximation method to simplify the mathematical analysis of the system and make it sensed biologically, whereas we transfer non-autonomous system that contain fluctuations terms to autonomous systems. The method leads to approximation the system to particular cases. However, some cases cannot be analyzed through this method, so the numerical simulations are used to present these cases. This method has been used by some researchers to analyze SIR systems [17, 18].

3.4 Piecewise function of sudden changes

Through the approximation method, we highlight two cases which represent the end points of $\in$ where$0 \leq \in \leq 1$, in other words, we consider the smallest and biggest values of the interference in degree of interaction through effects fluctuations. Therefore, we can describe these cases to represent sudden variation on prey predator system. With existence many impacts which affect on interactions of species suddenly. We use piecewise functions to describe these interactions.

The sinusoid function is represented through piecewise function as follows:

$$P(t) = \begin{cases} 0 & Bad \text{ scenarios} \\ 1 & Normal \text{ scenarios} \\ 2 & Good \text{ scenarios} \end{cases} \quad (7)$$

We aim through using piecewise function to describe different cases of fluctuations where the scenarios that represent sudden fluctuations on prey predator system are taken into consideration. The bad scenario means that surrounded circumstances are bad of prey predator interactions, in case the normal scenario means the surrounded circumstances are normal of prey predator interactions, while the good scenario means that surrounded circumstances are good of prey predator interactions.

So the systems become as follows:

The bad scenario: when we use $Q(t)$=0, the systems become as follows

In case the fluctuations in growth rate of prey, so the system becomes as follows:

$$\frac{dx}{dt} = -\frac{\alpha x}{1+h\alpha x}y,$$
$$\frac{dy}{dt} = -uy + \frac{e\alpha x}{1+h\alpha x}y - \frac{e\alpha}{(1+h\alpha x)}y^2 \quad (8)$$

In case the fluctuations in functional and numerical responses, we have the following system

$$\frac{dx}{dt} = x(1-x),$$
$$\frac{dy}{dt} = -uy - \frac{e\alpha}{(1+h\alpha x)}y^2 \quad (9)$$

The normal scenario: when using $Q(t)$=1 with growth rate of prey and with functional and numerical responses, the systems refer to original system (1).

The good scenario: when we use $Q(t)$=2, the systems become as follows

In case the fluctuations in growth rate of prey, so the system becomes as follows:

$$\frac{dx}{dt} = 2x(1-x) - \frac{\alpha x\, y}{1+h\alpha x},$$
$$\frac{dy}{dt} = -uy + \frac{e\alpha xy}{1+h\alpha x} - \frac{e\alpha}{(1+h\alpha x)}y^2, \quad (10)$$

In case the fluctuations in functional and numerical responses, we have the following system

$$\frac{dx}{dt} = x(1-x) - 2\frac{\alpha x\, y}{1+h\alpha x},$$
$$\frac{dy}{dt} = -uy + 2\frac{e\alpha xy}{1+h\alpha x} - \frac{e\alpha}{(1+h\alpha x)}y^2, \quad (11)$$

4. RESULTS AND DISCUSSION

We present the results through two sides; the first side is the theoretical side that involves studying effects fluctuations of sudden changes, so we explain effects fluctuations on Kolmogorov conditions. The second side represents numerical simulations, which they are used to study effects fluctuations on gradual changes.

4.1 Kolmogorov Conditions

Freedman, I. [19] summarized twelve conditions to represent Kolmogorov conditions. However, the most important condition is the condition that denote to existence the system (i.e. permanent coexistence prey with predator in the environment). Kolmogorov conditions are used to show that the system represents prey predator system. In addition to, these conditions are used to validate the values of parameters that are selected in numerical simulations in many prey predator studies.

Effects fluctuations on Kolmogorov conditions are explained of three scenarios as follows:

The bad scenario: when we use $Q(t)$=0 with growth rate of prey, the prey is going to extinct, this means that destroy prey predator system, so the Kolmogrov condition is not applicable and the system will not be as prey predator system. When using $Q(t)$=0 with functional and numerical responses; then the predator leaves the system (immigration or extinction) and prey arrive to carrying capacity, subsequently the system will not be as prey predator system and the Kolmogrov condition is not applicable.

The normal scenario: when we use $Q(t)$=1 with growth rate of prey and with functional and numerical responses, the system refer to system (1), the system is considered as prey predator system if the following condition is satisfied:

$$0 < \frac{u}{e\alpha - uh\alpha} < 1 \quad (12)$$

The good scenario: when we use $Q(t)$=2 with growth rate of prey, the system become system (10), the system is considered as prey predator system if the condition (12) is satisfied. When we use $Q(t)$=2 with functional and numerical responses, the system refer to system (11), the system is considered as prey predator system if the following condition is satisfied:

$$0 < \frac{u}{2e\alpha - uh\alpha} < 1 \quad (13)$$

We summarize the results of three scenarios in the following table:

Table1: summarized theoretical results of three scenarios

The case	The fluctuations with	The result	Kolmogrov condition
$Q(t)$=0	Prey equation	Destroyed all the system	Not Applicable
$Q(t)$=0	Prey and predator equations	The predator leave the system (immigration or extinction) and prey arrive to carrying capacity	Not Applicable
$Q(t)$=1	Prey equation	System (1)	$0 < \frac{u}{e\alpha - uh\alpha} < 1$
$Q(t)$=1	Prey and predator equations	System (1)	$0 < \frac{u}{e\alpha - uh\alpha} < 1$
$Q(t)$=2	Prey equation	System (10)	$0 < \frac{u}{e\alpha - uh\alpha} < 1$
$Q(t)$=2	Prey and predator equations	System (11)	$0 < \frac{u}{2e\alpha - uh\alpha} < 1$

4.2 Numerical Simulations

Non classical system and non autonomous differential equations are presented through the system (5), so some numerical simulations are performed to show effects the fluctuations on dynamical behaviors. Time series and phase space figures are used to present the dynamical behaviors. The values of parameters are selected to satisfy Kolmogrov condition (12) of the original system. The value of ϵ is varied to present effects the strength of fluctuations on prey predator system, while other parameters and initial conditions are fixed as follows:

$\alpha = 8.5$, $h = 0.5$, $u = 0.1$, $e = 0.2$

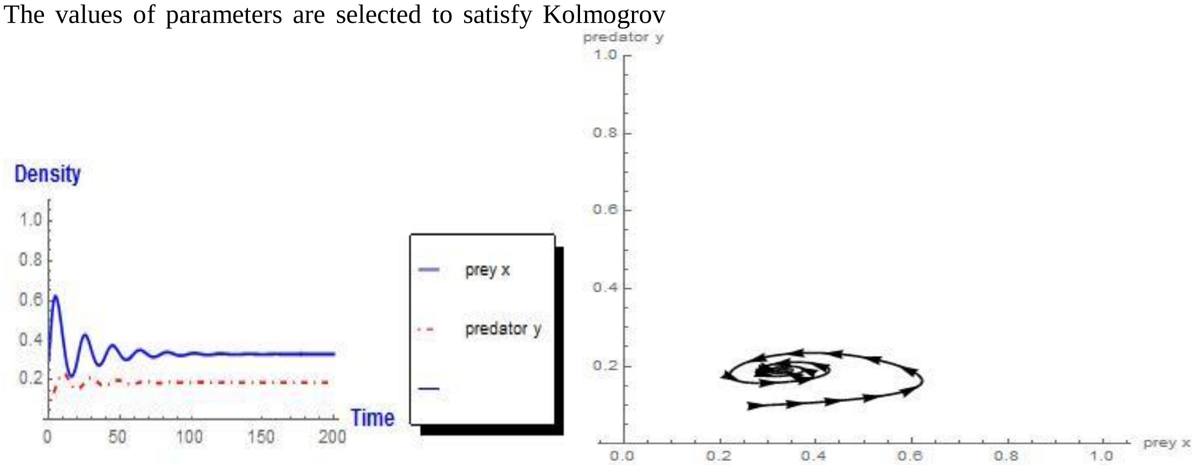

Figure 1: Dynamic behavior through investigating the formula (3) with functional and numerical responses when $= \mathbf{0.0}$: (a) Time series of the system (6) (b) Phase plane of the system (6).

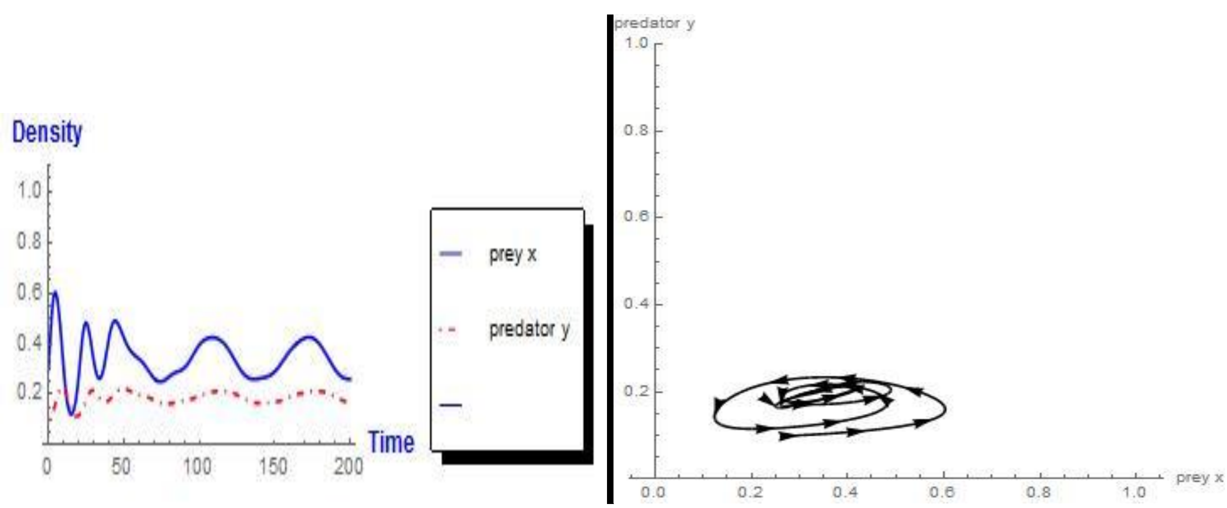

Figure 2: Dynamic behavior through investigating the formula (3) with functional and numerical responses when $= \mathbf{0.1}$: (a) Time series of the system (6) (b) Phase plane of the system (6).

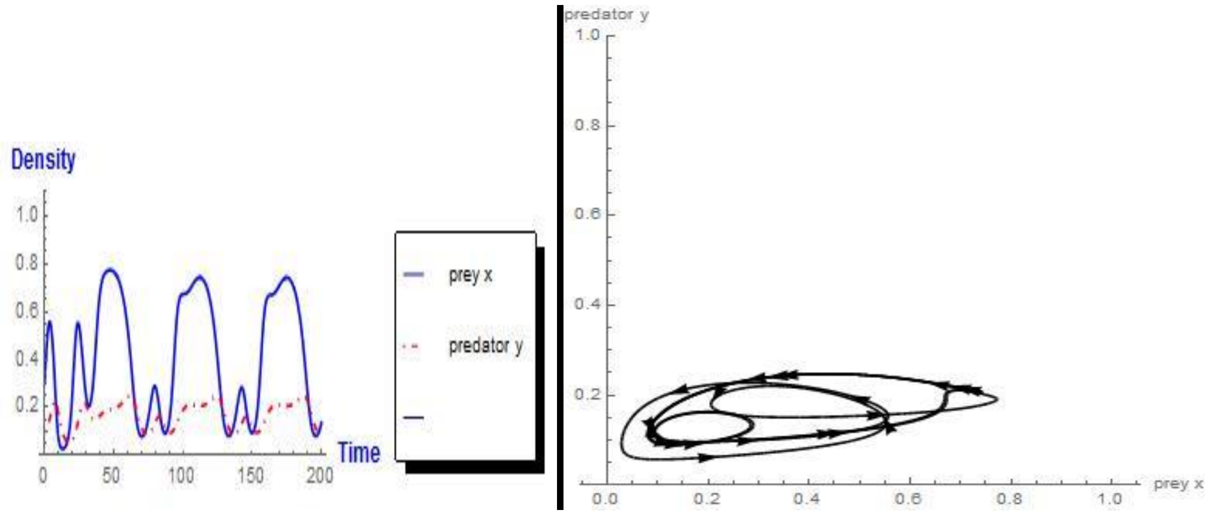

Figure 3: Dynamic behavior through investigating the formula (3) with functional and numerical responses when $= 0.4$: (a) Time series of the system (6) (b) Phase plane of the system (6).

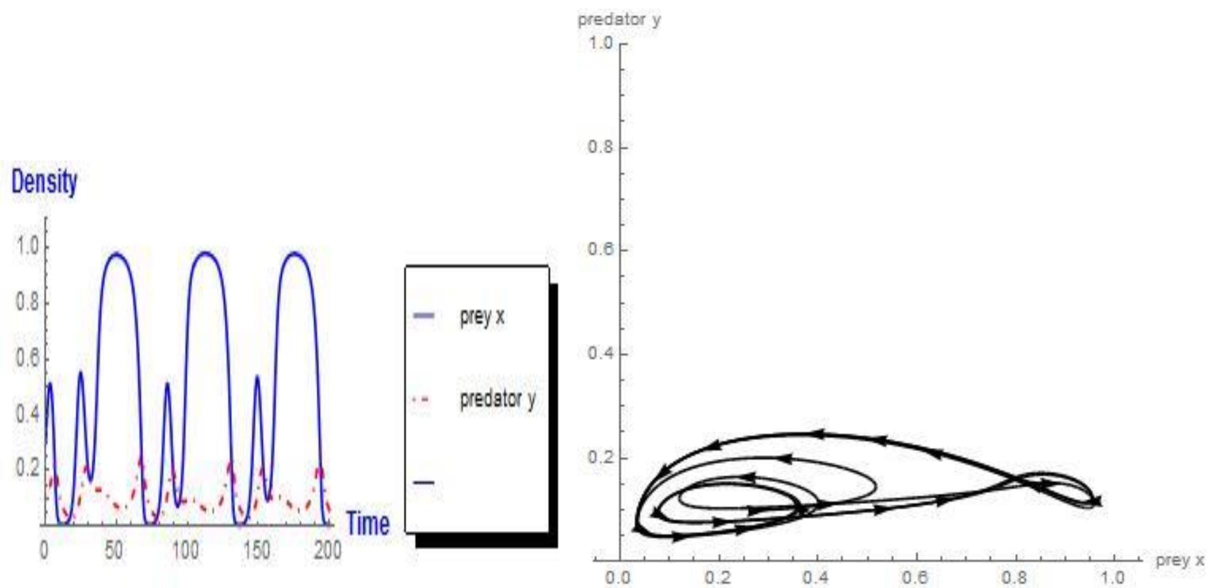

Figure 4: Dynamic behavior through investigating the formula (3) with functional and numerical responses when $= 0.8$: (a) Time series of the system (6) (b) Phase plane of the system (6).

We notice changing the dynamical behavior in each figure when increasing the value of ϵ through Figures 1, 2, 3 and 4. The changes in dynamical behavior involve changes the dynamical behaviors from stable case to periodic case when investigating the formula (3) in the system as shown through the Figure 1, which presents stable dynamical behavior of the original system(i.e when $\epsilon = 0$), while Figures 2,3, 4 present periodic dynamical behavior when investigating the formula (3) in the system. In addition, the changes in the dynamical behavior include increasing the size of limit cycle with increasing the value of ϵ as shown through Figures 2, 3, 4 respectively.

In the same manner, we conclude that the changes of dynamical behaviors are similar when investigating the formula (3) with growth rate of prey, but there is difference in the pattern of dynamical behaviors as shown through Figures 5,

Figures 6, 7.

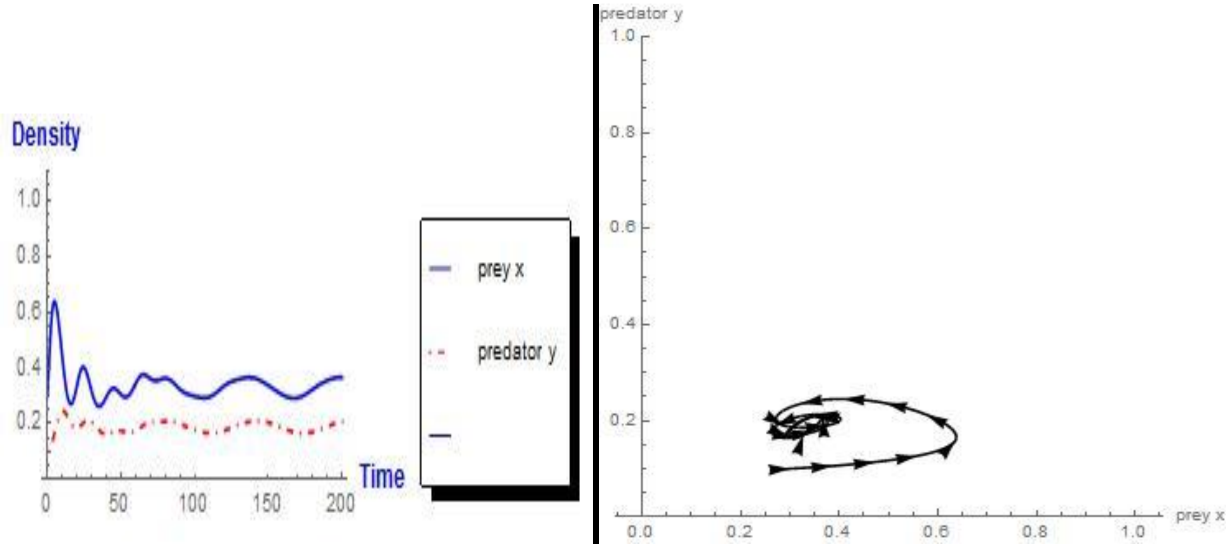

Figure 5: Dynamic behavior through investigating the formula (3) with growth rate of prey when $= \mathbf{0.1}$: (a) Time series of the system (5) (b) Phase plane of the system (5).

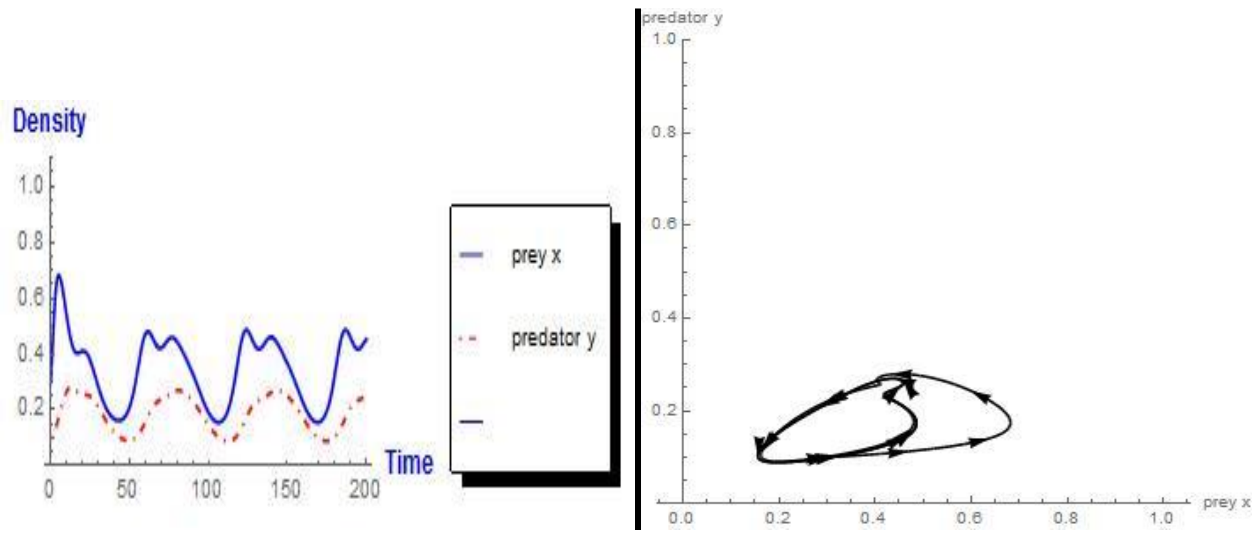

Figure 6: Dynamic behavior through investigating the formula (3) with growth rate of prey when $= \mathbf{0.4}$: (a) Time series of the system (5) (b) Phase plane of the system (5).

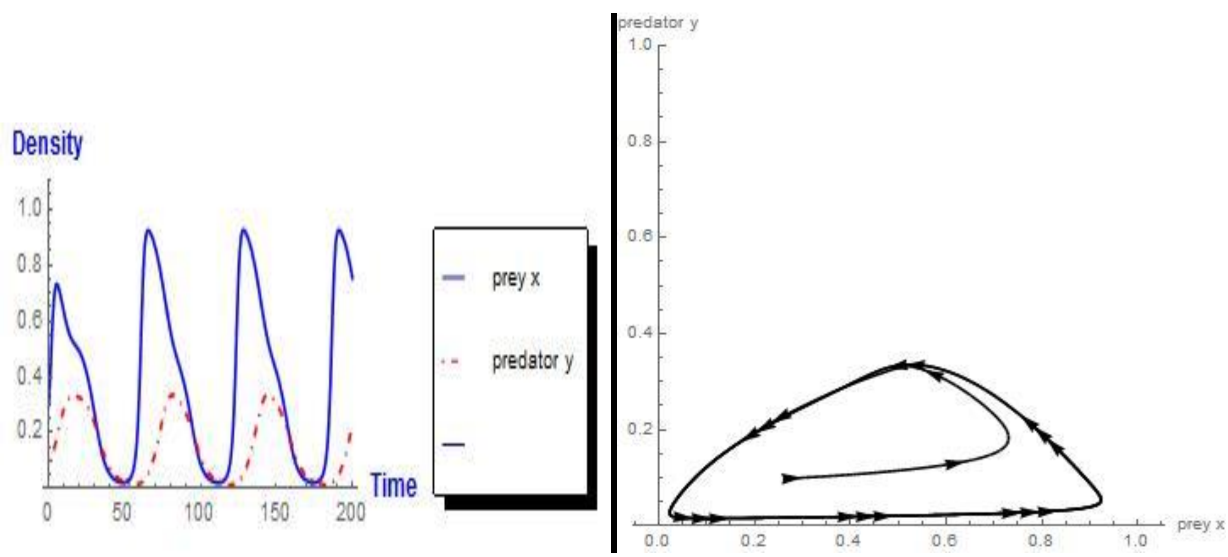

Figure 7: Dynamic behavior through investigating the formula (3) with growth rate of prey when $= \mathbf{0.8}$: (a) Time series of the system (5) (b) Phase plane of the system (5).

5. CONCLUSION

Fluctuations in prey predator system are taken into consideration to describe prey predator interactions, which make the description more realistic. The results are divided to theoretical and numerical sides. The theoretical results explain that Kolmogorov conditions change according to scenario in case normal and good scenarios, but Kolmogorov condition is not applicable of the bad scenario. Numerical simulation has been applied to explain effects of fluctuations on the dynamical behaviors of non-classical system, the numerical simulation show that the changes in dynamical behavior involve changes the dynamical behaviors from stable case to periodic case when investigating the sinusoid function in the system, and the changes in the dynamical behavior include changes the size of limit cycles, whereas the size of limit cycles increases with increasing the value of interference in degree of interaction ϵ. The same results are concluded when investigating the sinusoid function with growth rate of prey or with functional and numerical responses, but there is difference in the pattern of dynamical behaviors

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REFERENCES

[1] Y. Kuang, and E. Beretta, Global qualitative analysis of a ratio-dependentpredator-prey system, *Journal of Mathematical Biology* 36(4): 389–406, 1998.

[2] R. M. May, Limit cycles in predator-prey communities, *Science* 177: 900–902, 1972.

[3] J. Maynard-Smith, *Models in ecology*, CUP Archive, London, 1978.

[4] L. Perko, *Differential equations and dynamical Systems*, third edn, Springer, USA, 1996.

[5] J. Pastor, *Mathematical ecology of populations and ecosystems*, Wiley-Blackwell, USA, 2008.

[6] L. L. Rockwood, *Introduction to population ecology*, Cambridge Univ Press, 2006.

[7] J. D. Murray, *Mathematical biology*, Vol. 2, springer, New York, 2002.

[8] J. Alebraheem, , Y. Abu-Hassan. Dynamics of a Two Predator-One Prey System, Comp. Appl. Math. 2014 33:767–780, 2014.

[9] N. H. Gazi, and M. Bandyopadhyay, Effect of time delay on a harvested predator-prey model, Journal of Applied Mathematics and Computing 26(1): 263–280, 2008.

[10] R. Upadhyay, R.Naji, and N. Kumari Dynamical complexity in some ecological models: effect of toxin production by phytoplankton, Nonlinear Analysis: Modelling and Control 12(1): 123–138, 2007.

[11] EPA (2013). Climate change. Accessed: May 28, 2016.URL: https://www3.epa.gov/climatechange/impacts/

[12] R. K. Upadhyay, and S. Iyengar, Effect of seasonality on the dynamics of 2and 3 species prey-predator systems, *Nonlinear Analysis: Real World Applications*6(3): 509–530, 2007.

[13] S. Gakkhar, and R. K. Naji, Seasonally perturbed prey-predator system with predator-dependent functional response, *Chaos, Solitons & Fractals* 18(5): 1075–1083, 2003

[14] J. Alebraheem, , Y. Abu-Hassan, Simulation of Complex Dynamical Behavior in Prey Predator Model, International Conference on Statistics in Science, Business and Engineering 2012, Langkawi, Kedah, Malaysia, 10-12 September, 2012. **DOI:** 10.1109/ICSSBE.2012.6396618

[15] J. Alebraheem, Y. Abu-Hassan, Persistence of Predators in a Two Predators- One Prey Model with Non-Periodic Solution, Applied Mathematical Sciences, Vol. 6, No. 19, 943-956, 2012.

[16] H. Baek, and Y. Do, Seasonal effects on a beddington-deangelis type predator-prey system with impulsive perturbations, *Abstract and Applied Analysis* 2009 (2009), Article ID 695121, 19 pages, 2009. doi:10.1155/2009/695121.

[17] M. J.Keeling, P. Rohani, and B. T. Grenfell, Seasonally forced disease dynamics explored as switching between attractors, *Physica D: Nonlinear Phenomena* 148(3): 317–335, 2001.

[18] R. Olinky, A. Huppert, and L. Stone, Seasonal dynamics and thresholds governing recurrent epidemics, *Journal of Mathematical Biology* 56(6): 827–839, 2008.

[19] I.Freedman, *Deterministic mathematical models in population ecology*, Marcel dekker, inc, New York, 1980.

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