ON SOME PROPERTIES OF IDEAL BINARY SUPRA TOPOLOGICAL SPACE AND ΨB_S OPERATOR

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ABSTRACT: *In this paper, we introduce the concept of binary ideal in supra binary topological space and define the Ideal binary supra topological space. Also, we define two set operators, $\mathcal{B}_S$ –local function and operator and examine some of their properties. Finally, the concept of $\mathcal{B}_S$ –codense ideal is defined and investigated.*

Keywords: Ideal binary supra topological space, B_s –local function, ΨB_S operator, B_s –codense ideal

1 INTRODUCTION

Kuratowski [1] and Vaidyanathswamy [2], were the first to introduce the concept of the ideal in topological space. They also defined local function in ideal topological space. Hamlett and Jankovic in [3] and [4], studied the properties of ideal topological spaces and introduced an operator called Ψ operator. In 2014, Renukadevi *et al* [5], introduced the concepts of $\mu-$ local function, Ψ_μ operator, $\mu-$codense ideal, and $\mu-$ compatible ideal in ideal supra topological space.

The concept of binary topological space is a new idea in literature. It was introduced by S. Jothi and P. Thangavelu [6] in 2011, where the binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. In 2017, M. Thivagar and J. Kavitha [7], merged binary and supra topological space and formed a new topological structure called binary supra topological space. In 2018, Al-Omari and Modak [8], introduced another new concepts, binary ideal topological space, and binary local function. Also, they studded some generalized closed sets and characterized them.

In this work, we introduce the concept of binary ideal in supra binary topological space and define the Ideal binary supra topological space. Also, we define two sets of operators, B_S –local function and Ψ_{B_S} operator and discuss their properties. Finally, we introduce the B_S –codense ideal and examine the properties of this notions.

PRELIMINARIES AND DEFINITIONS

In this part, we will revisit essential definitions and their properties.

Definition 2.1. [5] Let (X,μ,I) be an ideal supra topological space. A set operator $(\,)^{*\mu}: K(X) \to K(X)$, is said to be $\mu-$ local function of I on X with respect to μ, and defined as:
$(E)^{*\mu}(I,\mu) = \{x \in X: U \cap E \notin I, for\ every\ U \in \mu(x)\}$,
where $\mu(x) = \{U \in \mu: x \in U\}$.

Definition 2.2 *[5] Let (X,μ,I) be an ideal supra topological space. An operator $\Psi\mu : K(X) \to \mu$ is defined for any $E \in K(X)$ by:*

$\Psi\mu(E) = \{x \in X : \exists\, U \in \mu(x): U - E \in I\}$ *and*
$\Psi\mu(E) = X - (X - E)^{*\mu}$

Definition 2.3 [5] *An ideal I in an ideal supra topological space (X,μ,I) is called $\mu-$ codense ideal if $\mu \cap I = \emptyset$.*

Definition 2.4 [7] *A binary supra topology from X to Y is a binary structure $B_s \subseteq K(X) \times K(Y)$, if the following holds:*

1. $(X,Y) \in B_s$ *and* $(\emptyset,\emptyset) \in B_s$.
2. $\{(E_\alpha, G_\alpha): \alpha \in \triangle\}$ *is a family of members of* B_s*, then* $(\cup E_\alpha, \cup G_\alpha) \in B_s$.

If B_s is a binary supra topology from X to Y then the triplet (X,Y,B_s) is called a binary supra topological space. The elements of B_s are called binary supra open sets and denoted by $bs-$open. The complement of binary supra open set is called binary supra closed and denoted by $bs-$closed.

Definition 2.5 [7] *Let (X,Y,B_s) be a binary supra topological space and let $(x,y) \in X \times Y$. A subset (E,G) of (X,Y) is called a binary supra neighborhood of (x,y) if there exist a binary supra open set (U,V) such that $(x,y) \in (U,V) \subseteq (E,G)$.*

Example 2.6 *Let $X = \{a,b,c\}$ and $Y = \{1,2\}$ with a binary supra topology*

$B_s = \{(X,Y), (\emptyset,\emptyset), (\{b\},\{1\}), (\{a,b\},\{2\}), (\{a,b\},Y)\}$.

Then, $(X,Y), (\{a,b\},\{2\}), (\{a,b\},Y)$ *is binary supra* *neighborhoods of the point* $(a,2)$.

Definition2.7 [7] *Let* (X,Y,B_s) *be a binary supra topological space and* $(E,G) \subseteq (X,Y)$. *The ordered pair* $((E,G)^{1^*},(E,G)^{2^*})$ is said to be a binary supra closure of (E,G), denoted by $Cl^{bs}(E,G)$ if:

1. $(E,G)^{1^*} = \cap\{E_\alpha : (E_\alpha, G_\alpha)$ *is* $bs-$ *closed and* $(E,G) \subseteq (E_\alpha, G_\alpha)\}$

.

$(E,G)^{2^*} = \cap\{B_\alpha : (E_\alpha, G_\alpha)$ *is* $bs-$ *closed and* $(E,G) \subseteq (E_\alpha, G_\alpha)\}$.

Definition 2.8 *[7] Let* (X,Y,B_s) *be a binary supra topological space and* $(E,G) \subseteq (X,Y)$. *The ordered pair* $((E,G)^{1^\circ},(E,G)^{2^\circ})$ *is called a binary supra interior of* (E,G), *denoted by* $Int^{bs}(E,G)$ *if:*

1. $(E,G)^{1^\circ} = \cup\{E_\alpha : (E_\alpha, G_\alpha)$ *is* $bs-$ *open and* $(E_\alpha, G_\alpha) \subset (E,G)\}$.
2. $(E,G)^{2^{\circ\circ}} = \cup\{G_\alpha : (E_\alpha, G_\alpha)$ *is* $bs-$ *open and* $(E_\alpha, G_\alpha) \subset (E,G)\}$.

Example 2.9 *Let* $X = \{a,b,c\}$ *and* $Y = \{1,2\}$ *with the binary supra topology*

Then, the $bs-$ *closed sets* are:

$B_s = \{(X,Y),(\emptyset,\emptyset),(\{a\},Y),(\{a,b\},\{2\}),(\emptyset,Y),(\{a,b\},Y)\}$.

$\{(X,Y),(\emptyset,\emptyset),(\{b,c\},\emptyset),(\{c\},\{1\}),(X,\emptyset),(\{c\},\emptyset)\}$. If $(E,G) = (\{b\},\emptyset)$ *then,* $Cl^{bs}(E,G) = (\{b,c\},\emptyset)$, and $Int^{bs}(E,G) = (\emptyset,\emptyset)$.

Theorem 2.10 *[7] Let* (X,Y,B_s) *be a binary supra topological space. For a subset* $(E,G) \subseteq (X,Y)$ *the following hold,*

1. $Cl^{bs}(E,G)$ *is the smallest* $bs-$*closed set containing* (E,G).
2. (E,G) *is a* $bs-$*closed in* (X,Y,B_s) *iff* $(E,G) = Cl^{bs}(E,G)$

Proposition 2.11 *[7] Let* (X,Y,B_s) *be a binary supra topological space and let* (E,G) *and* (H,L) *be subsets of* $K(X) \times K(Y)$. *Then*

1. $Cl^{bs}(\emptyset,\emptyset) = (\emptyset,\emptyset)$ *and* $Cl^{bs}(X,Y) = (X,Y)$,
2. $(E,G) \subseteq Cl^{bs}(E,G)$,
3. $Cl^{bs}(Cl^{bs}(E,G)) = Cl^{bs}(E,G)$,
4. $Cl^{bs}(E,G) \cup Cl^{bs}(H,L) \subseteq Cl^{bs}((E,G) \cup (H,L))$
5. $Cl^{bs}((E,G) \cap (H,L)) \subseteq Cl^{bs}(E,G) \cap Cl^{bs}(H,L)$.

Proposition 2.12 *[7] Let* (X,Y,B_s) *be a binary supra topological space. For a subset* $(E,G) \subseteq (X,Y)$ *the following hold,*

1. $Int^{bs}(E,G)$ *is a* $bs-$ *open.*
2. $Int^{bs}(E,G)$ *is the largest* Int^{bs} *open set contained in* (E,G).
3. (E,G) *is a* $bs-$*open if and only if* $Int^{bs}(E,G) = (E,G)$.

Theorem 2.13 *[7] Let* (X,Y,B_s) *be a binary supra topological space and let* (E,G) *and* (H,L) *be subsets of* $K(X) \times K(Y)$. *Then*

1. $Int^{bs}(\emptyset,\emptyset) = (\emptyset,\emptyset)$,
2. $Int^{bs}(X,Y) = (X,Y)$,
3. $Int^{bs}((E,G) \cap (H,L)) \subseteq Int^{bs}(E,G) \cap Int^{bs}(H,L)$,
4. $Int^{bs}(Int^{bs}(E,G)) = Int^{bs}((E,G)$,
5. $Int^{bs}((E,G) \cup Int^{bs}((H,L) \subseteq Int^{bs}((E,G) \cup (H,L))$.

Proposition 2.14 *Let* (X,Y,B_s) *be a binary supra topological space and let* (E,G) *be subset of* $K(X) \times K(Y)$. *Then*

1.
2. $(X,Y) - Int^{bs}(E,G) = Cl^{bs}((X,Y)-(E,G))$.

Proof.

1. Let $(x,y) \in (X,Y) - Cl^{bs}(E,G)$. Since $(X,Y) - Cl^{bs}((E,G) = Int^{bs}((X,Y)-(E,G))$, $(X,Y) - Cl^{bs}(E,G)$ is a $bs-$open set and $(X,Y) - Cl^{bs}(E,G) \subseteq ((X,Y)-(E,G))$, then Hence,

$$(X,Y) - Cl^{bs}((E,G) \subseteq Int^{bs}((X,Y)-(E,G))\ldots\ldots(i)$$

Conversely, Let $(x,y) \in Int^{bs}((X,Y)-(E,G))$, then,

$$(x,y) \in Int^{bs}((X,Y)-(E,G))$$

$$Int^{bs}((X,Y)-(E,G)) \cap (E,G) = \emptyset.$$

This implies that $(x,y) \notin (E,G)$, and hence $(x,y) \notin (E,G) \cap (U,V)$ for any $bs-$open set containing (x,y). Therefore, $(x,y) \notin Cl^{bs}(E,G)$, and then $(x,y) \in (X,Y) - Cl^{bs}(E,G)$. Hence,

$Int^{bs}((X,Y)-(E,G))$

$\subseteq (X,Y) - Cl^{bs}(E,G)$……$(ii)$

By (i) and (ii) the proof is complete.

2. By (1) we have $(X,Y) - Cl^{bs}(E,G) = Int^{bs}((X,Y) - (E,G))$, then

$(X,Y) - Cl^{bs}((X,Y) - (E,G)) = Int^{bs}(E,G)$.

Taking complements of both sides implies:

$Cl^{bs}((X,Y) - (E,G)) = (X,Y) - Int^{bs}(E,G)$.

Definition 2.15 *[8] Let X and Y be any two non-empty sets. A binary ideal from X to Y is a binary structure $\boldsymbol{I} \subseteq K(X) \times K(Y)$ that satisfies the following axioms:*

1. $(E,G) \in \boldsymbol{I}$ *and* $(H,L) \subseteq (E,G)$ *implies* $(H,L) \in \boldsymbol{I}$.
2. $(E_1,G_1) \in \boldsymbol{I}$ *and* $(E_2,G_2) \in \boldsymbol{I}$ *implies:* $(E_1 \cup E_2, G_1 \cup G_2) \in \boldsymbol{I}$

2 Ideal binary supra topological space

In this part, we introduce the concepts of ideal binary supra topological space and binary $s-$ local function and discuss some of their properties.

Definition 3.1 A binary supra topological space (X,Y,B_s) with a binary ideal $\boldsymbol{I}$ on $K(X) \times K(Y)$ is called an ideal binary supra topological space and is denoted by $(X,Y,B_s,\boldsymbol{I})$.

Definition 3.2 *Let $(X,Y,B_s,\boldsymbol{I})$ be an ideal binary supra topological space. A set operator $(,)^{*s} : K(X) \times K(Y) \to K(X) \times K(Y)$, is called a binary $s-$ local function, and is defined as:*

$(E,G)^{*s}(\boldsymbol{I},B_s) = \{(x,y) \in (X,Y) : (U \cap E, V \cap G) \notin \boldsymbol{I}$ *for every* $(U,V) \in B_s(x,y)\}$ *where,*

$B_s(x;y) = \{(U,V) \in B_s : (x,y) \in (U,V)\}$.

Theorem 3.3 *For a space $(X,Y,Bs,\boldsymbol{I})$, let $(E,G),(H,L),(E_1,G_1),(E_2,G_2)\cdots(E_i,G_i) \subset X \times Y$. Then,*

1. $(\emptyset,\emptyset)^{*s} = (\emptyset,\emptyset)$.
2. $(E,G) \subset (H,L) \Rightarrow (E,G)^{*s} \subset (H,L)^{*s}$
3. *For another ideal* $\boldsymbol{J} \supseteq \boldsymbol{I}$ *on* $X \times Y$, $(E,G)^{*s}(\boldsymbol{J}) \subset (E,G)^{*s}(\boldsymbol{I})$.
4. $(E,G)^{*s} \subset Cl^{bs}(E,G)$,
5. $(E,G)^{*s}$ *is a $bs-$closed set,*
6. $((E,G)^{*s})^{*s} \subset (E,G)^{*s}$,
7. $(E,G)^{*s} \cup (H,L)^{*s} \subset ((E,G) \cup (H,L))^{*s}$,
8. $\cup_i((E,G)^{*s} \subset (\cup_i (E_i,G_i))^{*s}$,
9. $((E,G) \cap (H,L))^{*s} \subset (E,G)^{*s} \cap (H,L)^{*s}$,
10. *For* $(U,V) \in B_s, (U,V) \cap ((U,V) \cap (E,G))^{*s} \subset (U,V) \cap (E,G)^{*s}$,
11. *For* $I \in \boldsymbol{I}, ((E,G) \cup I)^{*s} = (E,G)^{*s} = ((E,G) - I)^{*s}$

Proof.

1. Obvious by the definition of binary $s-$local function.

2. Let $(E,G) \subset (H,L)$ and $(x,y) \in (E,G)^{*s}$. Then for every $(U,V) \in B_s(x,y), (U \cap E, V \cap G) \notin \boldsymbol{I}$. Since, $(U \cap E, V \cap G) \subset (U \cap H, V \cap L)$, then $(U \cap H, V \cap L) \notin \boldsymbol{I}$. This implies that $(x,y) \in (H,L)^{*s}$.

3. Let $\boldsymbol{J} \supseteq \boldsymbol{I}$ and $(x,y) \in (E,G)^{*s}(\boldsymbol{J})$. Then for every $(U,V) \in B_s(x,y), (U \cap E, V \cap G)$. So, $(U \cap E, V \cap G) \notin \boldsymbol{I}$. Hence, $(x,y) \in (E,G)^{*s}(\boldsymbol{I})$. Therefore, $(E,G)^{*s}(\boldsymbol{J}) \subset (E,G)^{*s}(\boldsymbol{I})$.

4. Let $(x,y) \in (E,G)^{*s}$. Then for every $(U,V) \in B_s(x,y), (U \cap E, V \cap G) \notin I$. Hence, $((U \cap E, V \cap G)) \neq (\emptyset,\emptyset)$, and $(x,y) \in Cl^{bs}(E,G)$. Therefore, $(E,G)^{*s} \subset Cl^{bs}(E,G)$.

5. To show that $(E,G)^{*s}$ is a $bs-$closed set and since each binary supra neighbourhood (μ,ν) of (x,y) contains a $(U,V) \in B_{s(x,y)}$, let $(E \cap \mu, G \cap \nu) \in \boldsymbol{I}$, then for $(E \cap U, G \cap V) \subset (E \cap \mu, G \cap \nu), (E \cap U, G \cap V) \in \boldsymbol{I}$. It follows that, $(X,Y) - ((E,G)^{*s})$ is a union of $bs-$open sets. Since the arbitrary union of $bs-$open sets

is a $bs-$open set, then $(X,Y)-(E,G)^{*s}$ is a $bs-$open set and hence $(E,G)^{*s}$ is a $bs-$closed set.

6. By (4), $((E,G)^{*s})^{*s} \subset \left(Cl^{bs}(E,G)\right)^{*s} = (E,G)^{*s}$ since, $(E,G)^{*s}$ is a $bs-$closed set by (5).

7. For the subsets $(E,G),(H,L)$, we have $(E,G) \subset (E\cup H, G\cup L)$ and $(H,L) \subset (E\cup H, G\cup L)$, so, by (2), $(E,G)^{*s} \subset (E\cup H, G\cup L)^{*s}$, and, $(H,L)^{*s} \subset (E\cup H, G\cup L)^{*s}$. Therefore, $(E,G)^{*s} \cup (H,L)^{*s} \subset (E\cup H, G\cup L)^{*s} = ((E,G)(H,L))^{*s}$

. 8. Directly by (7).

9. For the subsets $(E,G),(H,L)$, we have $(E\cap H, G\cap L) \subset (E,G)$, and $(E\cap H, G\cap L) \subset (H,L)$. Then, by (2), $(E\cap H, G\cap L)^{*s} \subset (E,G)^{*s}$, and $(E\cap H, G\cap L)^{*s} \subset (H,L)^{*s}$. Therefore, $\left((E,G)\cap(H,L)\right)^{*s} = (E\cap H, G\cap L)^{*s} \subset (E,G)^{*s} \cap (H,L)^{*s}$.

10. Since, $(U,V)\cap(E,G) \subset (E,G)$, then, $\left((U,V)\cap(E,G)\right)^{*s} \subset (E,G)^{*s}$, and hence, $(U,V) \cap ((U,V)\cap(E,G)^{*s} \subset (U,V)\cap(E,G)^{*s}$.

1. Since, $(E,G) \subset \left((E,G)\cup I\right)$, then, by (2),

$$(E,G)^{*s} \subset \left((E,G)\cup I\right)^{*s} \ldots\ldots\ldots(i)$$

Let $(x,y) \in \left((E,G)\cup I\right)^{*s}$. Then, for every $(U,V) \in B_s(x,y), (U,V)\cap\left((E,G)\cup I\right) \notin \boldsymbol{I}$. Hence, $(U,V)\cap(E,G) \notin \boldsymbol{I}$. Assume that $(U,V)\cap(E,G) \in I$. Since $(U,V)\cap I \subset I$ implies $(U,V)\cap I \subset \boldsymbol{I}$, then $(U,V)\cap\left((E,G)\cup I\right) \in \boldsymbol{I}$, which is a contradiction. Hence $(x,y) \in (E,G)^{*s}$ and,

$$((E,G)\cup I)*^{s} \subset (E,G)^{*s} \ldots\ldots\ldots(ii)$$

From (i) and (ii) we have

$$\left((E,G)\cup I\right)^{*s} = (E,G)^{*s} \ldots\ldots\ldots(iii)$$

Now, Since $\left((E,G)-I\right) \subset (E,G)$, then,

$$\left((E,G)-I\right)^{*s} \subset (E,G)^{*s} \ldots\ldots\ldots(iv)$$

Finally, let $(x,y) \in (E,G)^{*s}$. We claim that $(x,y) \in (E-I)^{*s}$. Suppose this is not the case, then there is a $(U,V) \in B_s(x,y)$ such that $(U,V)\cap\left((E,G)-I\right) \in \boldsymbol{I}$. Let $I \in \boldsymbol{I}$, then $I \cup \left((U,V)\cap\left((E,G)-I\right)\right) \in \boldsymbol{I}$. This implies that $I \cup ((U,V)\cap((E,G) \in \boldsymbol{I}$. Hence, ($U,V)\cap((E,G) \in \boldsymbol{I}$, which is a contradiction to the fact that $(x,y) \in (E,G)^{*s}$. Therefore,

$$(E,G)^{*s} \subset \left((E,G)-I\right)^{*s} \ldots\ldots\ldots(v)$$

Hence, from (iii), (iv) and (v) we have

$$\left((E,G)\cup I\right)^{*s} = (E,G)^{*s} = \left((E,G)-I\right)^{*s}$$

Following two examples, the first one supports the item (11) of Theorem 3.3, and the second one is a counterexample, showing that replacing $(\subset)$ with $(=)$ in item (10) of Theorem 3.3 is not hold in general.

Example 3.4 *Let* $X = \{a,b,c\}$ *and* $Y = \{1,2\}$ *with*

$$B_s = \{(X,Y),(\emptyset,\emptyset),(\{a\},Y),(\{a,b\},\{2\}),(\emptyset,Y),(\{a,b\},Y)\}$$

and a binary ideal $\boldsymbol{I} = \{(\emptyset,\emptyset),(\{b\},\emptyset)\}$. *Let* $I \in \boldsymbol{I}$, *where* $I = (\{b\},\emptyset)$ *and* $(H,L) = (\{c\},\emptyset)$. *We show that* $\left((H,L)\cup(I)\right)^{*s} = (H,L)^{*s}$. *The* $bs-$*open sets containing*

$(a,1)$ are: $\{(X,Y),(\{a\},Y),(\{a,b\},Y)\}$,

$(a,2)$ are: $\{(X,Y),(\{a\},Y),(\{a,b\},\{2\}),(\{a,b\},Y)\}$,

$(b,1)$ are: $\{(X,Y),(\{a,b\},Y)\}$

$(b,2)$ are: $\{(X,Y),(\{a,b\},\{2\}),(\{a,b\},Y)\}$,

$(c,1)$ is: (X,Y),

$(c,2)$ is: (X,Y),

Now,

$$(H,L)^{*s} = \{(c,1),(c,2)\} \ldots\ldots\ldots(1)$$

while, $(H,L)\cup I = (\{b,c\},\emptyset)$, and

$$\left((H,L)\cup I\right)^{*s} = \{(c,1),(c,2)\} \ldots\ldots\ldots(2)$$

Hence, from (1) and (2), we get

$((H,L)\cup I)^{*s} = (H,L)^{*s}$

Example 3.5 *Let* $X = \{a,b,c,d\}$ *and* $Y = \{1,2\}$ *with*

$B_s = \{(X,Y),(\emptyset,\emptyset),(\{a\},Y),(\{a,b\},\{2\}),(\emptyset,Y),(\{a,$

and a binary ideal $\boldsymbol{I} = \{(\emptyset,\emptyset),(\{c\},\emptyset)\}$. *Let* $(E,G) = (\{b\},\emptyset)$ *and* $(H,L) = (\emptyset,\{1\})$, *then, the* $bs-$ *open sets containing*

$(a,1)$ are: $\{(X,Y),(\{a\},Y),(\{a,b\},Y)\}$,
$(a,2)$ are: $\{(X,Y),(\{a\},Y),(\{a,b\},\{2\}),(\{a,b\},Y)\}$,

$(b,1)$ are: $\{(X,Y),(\{a,b\},Y)\}$

$(b,2)$ are: $\{(X,Y),(\{a,b\},\{2\}),(\{a,b\},Y)\}$,
$(c,1)$ is: (X,Y),
$(c,2)$ is: (X,Y),
$(d,1)$ is: (X,Y),
$(d,2)$ is: (X,Y).
Now,

$(E,G)^{*s} = \{(b,1),(b,2),(c,1),(c,2),(d,1),$

and

$(H,L)^{*s} = \{(a,1),(b,1),(c,1),(c,2),(d,1),(d,2)\}$

then,

$(E,G)^{*s}\cup(H,L)^{*s} =$
$\{(a,1),(b,1),(b,2),(c,1),(c,2),(d,1),(d,2)\}$...(1)

while $(E,G)\cup(H,L) = (\{b\},\{1\})$ and

$((E,G)\cup(H,L))^{*s} = \{(a,1),(a,2),(b,1),(b,2),(c,$

Hence, from (1) and (2), we get

$(E,G)^{*s}\cup(H,L)^{*s} \neq ((E,G)\cup(H,L))^{*s}$

3 Binary Supra Operator Ψ_{B_s}

In this section, we will introduce the concept of Binary Supra Operator Ψ_{B_s} and examine some of its properties.

Definition 4.1 *Let* $(X,Y,B_s,\boldsymbol{I})$ *be an ideal binary supra topological space. The binary supra operator* $\Psi_{B_s}: K(X)\times K(Y)\to B_s$ *is defined for every* $(E,G)\in K(X)\times K(Y)$ by $\Psi_{B_s}(E,G) = \{(x,y)\in(X,Y): \exists\,(U,V)\in B_s(x,y)$ *such that* $(U,V)-(E,G)\in\boldsymbol{I}\}$, *and*

$\Psi_{B_s}(E,G) = (X,Y) - ((X,Y)-(E,G))^{*s}$

Theorem 4.2 *Let* $(X,Y,B_s,\boldsymbol{I})$ *be an ideal binary supra topological space. Then*

1. *If* $(E,G)\subseteq(X,Y)$, *then* $\Psi_{B_s}(E,G)$ *is a* $bs-$ *open.*
2. *If* $(E,G)\subseteq(H,L)$, *then* $\Psi_{B_s}(E,G)\subseteq\Psi_{B_s}(H,L)$.
3. *If* $(E,G),(H,L)\in K(X)\times K(Y)$, *then* $\Psi_{B_s}(E,G)\cup\Psi_{B_s}(H,L)\subset\Psi_{B_s}((E,G)\cup(H,L))$
4. *If* $(E,G),(H,L)\in K(X)\times K(Y)$, *then* $\Psi_{B_s}((E,G)\cap(H,L))\subset\Psi_{B_s}(E,G)\cap\Psi_{B_s}(H,L)$
5. *If* $(U,V)\in B_s$, *then* $(U,V)\subset\Psi_{B_s}(U,V)$.
6. *If* $(E,G)\subseteq(X,Y)$, *then* $\Psi_{B_s}(E,G)\subset\Psi_{B_s}(\Psi_{B_s}((E,G))$
7. *If* $(E,G)\in\boldsymbol{I}$, *then* $\Psi_{B_s}(E,G) = (X,Y)-(X,Y)^{*s}$
8. *If* $(E,G)\subseteq(X,Y)$, *then* $Int^{bs}(E,G)\subset\Psi_{B_s}(E,G)$.
9. *If* $(E,G)\subseteq(X,Y)$, *then* $((X,Y)-(E,G))^{*s} = ((X,Y)-(E,G)^{*s})^{*s}$ $\Psi_{B_s}(E,G) = \Psi_{B_s}(\Psi_{B_s}(E,G))$ *if and only if*
10. *If* $(E,G)\subseteq(X,Y), I\in\boldsymbol{I}$, *then* $\Psi_{B_s}((E,G)-I) = \Psi_{B_s}(E,G)$.
11. *If* $(E,G)\subseteq(X,Y), I\in\boldsymbol{I}$, *then* $\Psi_{B_s}((E,G)\cup I) = \Psi_{B_s}(E,G)$.
12. *If* $((E,G)-(H,L))\cup((H,L)-(E,G))\in\boldsymbol{I}$, *then* $\Psi_{B_s}(E,G) = \Psi_{B_s}(H,L)$.

Proof.

1. We know that $((X,Y)-(E,G))^{*s}$ is a $bs-$ closed by (5) of Theorem 3.3. Then $(X,Y)-((X,Y)-(E,G))^{*s}$ is a $bs-$open set. Hence, $\Psi_{B_s}(E,G)$ is a $bs-$ open.

2. Let $(E,G) \subseteq (H,L)$, then $((X,Y)-(H,L)) \subset ((X,Y)-(E,G))$. By (2) of Theorem3.3,

$((X,Y)-(H,L))^{*s} \subset ((X,Y)-(E,G))^{*s}$. Therefore,

$\Psi_{B_s}(E,G) \subseteq \Psi_{B_s}(H,L)$.

3. Since $(E,G) \subset (E \cup H, G \cup L)$, and $(H,L) \subset (E \cup H, G \cup L)$, then, $(E,G) \cup (H,L) \subset (E \cup H, G \cup L)$. Hence, by (2),

$\Psi_{B_s}(E,G) \cup \Psi_{B_s}(H,L) \subset \Psi_{B_s}((E,G) \cup (H,L))$.

4. Since $(E,G) \cap (H,L) \subset (E,G)$ and $(E,G) \cap (H,L) \subset (H,L)$, from (3), $\Psi_{B_s}((E,G) \cap (H,L)) \subset \Psi_{B_s}(E,G) \cap \Psi Bs(H,L)$.

5. Let $(U,V) \in B_s$. Then $((X,Y)-(U,V))$ is a $bs-$closed set and hence, $Cl^{bs}((X,Y)-(U,V)) = ((X,Y)-(U,V))$. By (4) of Theorem 3.3,

$((X,Y)-(U,V))^{*s} \subset Cl^{bs}((X,Y)-(U,V)) = ((X,Y)-(U,V))$

. Hence, $(U,V) \subset (X,Y) - ((X,Y)-(U,V))^{*s}$ and then, $(U,V) \subset \Psi_{B_s}(U,V)$.

6. By (1), $\Psi_{B_s}(E,G) \subset B_s$, and by (5) we have, $\Psi_{B_s}(E,G) \subset \Psi_{B_s}(\Psi_{B_s}((E,G))$.

7. Let $(E, G) \in$ **I**. Since, $\Psi_{B_s}(E,G) = (X,Y)-((X,Y)-(E,G))^{*s}$,and

$((X,Y)-(E,G))^{*s} = (X,Y)^{*s}$ by (11) of Theorem 3.3, then

$\Psi_{B_s}(E,G) = (X,Y)-(X,Y)^{*s}$.

8. Since for any $(E,G) \in K(X) \times K(Y)$,

$\Psi_{B_s}(E,G) = (X,Y)-((X,Y)-(E,G))^{*s}$

then by (4) of Theorem 3.3, we have,

$(X,Y) - Cl^{bs}((X,Y)-(E,G)) \subset (X,Y)-((X,Y)-(E$

and by (2) of Proposition 2.15, since

$(X,Y) - Int^{bs}(E,G) = Cl^{bs}((X,Y)-(E,G))$,

i.e.,

$Int^{bs}(E,G) = (X,Y) - Cl^{bs}((X,Y)-(E,G))$

Hence,

$Int^{bs}(E,G) \subset (X,Y)-((X,Y)-(E,G))^{*s} = \Psi_{B_s}((E,G)$

Therefore, $Intbs(E,G) \subset \Psi_{B_s}((E,G)$.

9. Let $\Psi_{B_s}(E,G) = \Psi_{B_s}(\Psi_{B_s}(E,G))$. Then,
This implies that,

$((X,Y)-(E,G))^{*s} = ((X,Y)-(E,G)^{*s})^{*s}$

$(X,Y) - ((X,Y)-(E,G))^{*s} = \Psi_{B_s}((X,Y) - ((X,Y)-(E,G))^{*s}) = (X,Y) - ((X,Y)-(E,G)^{*s})^{*s}$

10. By the definition of Ψ_{B_s} and by (11) of Theorem 3.3, we have,

$\Psi_{B_s}((E,G)-I) = (X,Y) - ((X,Y) - ((E,G)-I))^{*s}$
$= (X,Y)-(((X,Y)-(E,G)) \cup I))^{*s}$
$= (X,Y)-((X,Y)-(E,G))^{*s}$

Hence,

$\Psi_{B_s}((E,G)-I) = (X,Y)-((X,Y)-(E,G))^{*s} = \Psi_{B_s}(E,G)$

Therefore, $\Psi_{B_s}((E,G)-I) = \Psi_{B_s}(E,G)$.

11. By the definition of Ψ_{B_s} and by (11) of Theorem 3.3, we have,

$\Psi_{B_s}((E,G) \cup I)$
$= (X,Y) - ((X,Y) - ((E,G) \cup I))^{*s}$
$= (X,Y)-((((X,Y)-(E,G)) - I)^{*s}$
$= (X,Y)-((X,Y)-(E,G))^{*s}$

Hence,

$X,Y)-((X,Y)-(E,G))^{*s}$
$_{B_s}(E,G)$

Therefore, $\Psi_{B_s}((E,G) \cup I) = \Psi_{B_s}(E,G)$.

12. Let $((E,G)-(H,L)) \cup ((H,L)-(E,G)) \in \boldsymbol{I}$, and let $(E,G)-(H,L) = I_1, (H,L)-(E,G) = I_2$.

It is clear that I_1 and $I_2 \in \boldsymbol{I}$. Also observe that

$(H,L) = ((E,G)-I_1) \cup I_2 \ldots\ldots(i)$

by (11), $\Psi_{B_s}(E,G) = \Psi_{B_s}((E,G)-I_1) = \Psi_{B_s}((E,G)-I_1) \cup I_2)$. Hence, form (i) we have

$\Psi_{B_s}(H,L) = \Psi_{B_s}((E,G)-I_1) \cup I_2 = \Psi_{B_s}(E,G)$

.5. Binary Supra Codense Ideal

In this part, we introduce and define the concepts of $B_s -$ codense ideal, and investigate its properties.

Definition 5.1 *Let* $(X, Y, B_s, \boldsymbol{I})$ *be an ideal binary supra topological space. The binary ideal* $\boldsymbol{I}$ *is said to be a binary supra codense Ideal (briefly,* $B_s -$ *codense ideal) if* $B_s \cap \boldsymbol{I} = \{\emptyset, \emptyset\}$.

Theorem 5.2 *Let* $(X, Y, B_s, \boldsymbol{I})$ *be an ideal binary supra topological space, and* $\boldsymbol{I}$ *is a* $B_s -$*codense with* B_s*. Then* $(X, Y) = (X, Y)^{*s}$.

Proof. It is sufficient to show that $(X, Y) \subseteq (X, Y)^{*s}$ since $(X, Y)^{*s} \subseteq (X, Y)$ is clear. Let $(x, y) \in (X, Y)$ but $(x, y) \notin (X, Y)^{*s}$. Then, there exists $(U, V)_{(x,y)} \in B_s$ such that $(U, V)_{(x,y)} \cap (X, Y) \in \boldsymbol{I}$. This implies that, $(U, V)_{(x,y)} \in \boldsymbol{I}$, which is a contradiction since $(U, V)_{(x,y)} \cap \boldsymbol{I} = \{\emptyset, \emptyset\}$. Hence, $(x, y) \in (X, Y)^{*s}$, and then $(X, Y) \subseteq (X, Y)^{*s}$. Therefore, $(X, Y) = (X, Y)^{*s}$.

Theorem 5.3 *Let* $(X, Y, B_s, \boldsymbol{I})$ *be an ideal binary supra topological space. Then following conditions are equivalent:*

1. $B_s \cap \boldsymbol{I} = \{\emptyset, \emptyset\}$.

2. $\Psi_{B_s}(\emptyset, \emptyset) = (\emptyset, \emptyset)$.

3. *If* $I \in \boldsymbol{I}$*, then* $\Psi Bs(I) = (\emptyset, \emptyset)$.

Proof. (1) ⇒ (2), let $B_s \cap \boldsymbol{I} = \{\emptyset, \emptyset\}$. By Theorem 5.2, we have

$$\Psi_{B_s}(\emptyset, \emptyset) = (X, Y) - \left((X, Y) - (\emptyset, \emptyset)\right)^{*s} = (X, Y) - (X, Y)^{*s} = (\emptyset, \emptyset).$$

(2) ⇒ (3), let $I \in \boldsymbol{I}$. Since, $\Psi_{B_s}(I) = (X, Y) - ((X, Y) - I)^{*s}$, then by (11) of Theorem 3.3,

$$\left((X, Y) - I\right)^{*s} = (X, Y)^{*s}$$

and hence, $\Psi_{B_s}(I) = (X, Y) - (X, Y)^{*s} = (\emptyset, \emptyset)$ by Theorem 5.2.

(3) ⇒ (1), Let $(E, G) \in B_s \cap \boldsymbol{I}$, then $(E, G) \in \boldsymbol{I}$, and by (3), $\Psi_{B_s}((E, G)) = (\emptyset, \emptyset)$. Also, $(E, G) \in B_s$ and then by (5) of Theorem 4.2, *we* have $(E, G) \subset \Psi_{B_s}((E, G)) = (\emptyset, \emptyset)$. Hence, $B_s \cap \boldsymbol{I} = \{\emptyset, \emptyset\}$.

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