MICROPOLAR FLUIDS FLOW OVER A SHRINKING POROUS SURFACE IN THE PRESENCE OF MAGNETIC FIELD AND THERMAL RADIATION

Hassan Waqas[1], Sajjad Hussain[2, a], Abdul Saboor[3], Shamila Khalid[4]
syedhassanwaqas@hotmail.com[1], s.nawaz@mu.edu.sa[2], saboorfsd@hotmail.com[3], gcuflc@yahoo.com[4]
[1, 3,4]Department of Mathematics, Chanab College of Advance Studies, Faisal Abad, Pakistan
[1, 4] Presently Department of Mathematics, Govt College University Faisal Abad (Layyah Campus), Pakistan
[2]Presently Mathematics Department, Majmaah University, College of Science, Alzulfi, KSA
[2]Punjab Higher Education Department, College wing, Lahore, Pakistan
a: (Corresponding author) sajjadgut@gmail.com

ABSTRACT: *Micropolar fluids flow over a shrinking porous surface in the presence of magnetic field and thermal radiation is considered for mathematical analysis and numerical solution. The parametric study of the problem demonstrates the effects of the physical quantities like magnetic field strength, suction at the boundary, micopolarity of the fluid and radiative heat source. The mathematical model of the problem is transformed to its non- dimensional form by using similarity transforms to obtain the numerical solution. The results have been obtained for several representative values of the material parameters d_1, d_2 and d_3, shrinking surface parameter λ, magnetic parameter H, suction/injection parameter A, Eckert number E_c, Radiation parameter R_n and Prandtl number P_r. The results for non dimensional velocity, microrotation and temperature distribution are presented graphically.*

Key Words: Micropolar fluids, Shrinking porous surface, Hartmann number, Eckert number, Radiation parameter, Prandtl number.

1. INTRODUCTION

The micropolar fluid theory pioneered by Eringen [1] presents relatively a new research field. This model besides the generalization Navier-Stokes model takes into account the conservation of angular momentum due to local micromotion of the fluid particles. Researchers are engaged to explore innovative results related to micropolar fluids flow problems. Magyari *et al.* [2] considered the flow of micropolar fluids and solved a problem by Laplace transformation. Takhar *et al*. [3] presented the finite element solution for micropolar fluid flow and heat transfer for two porous discs. MHD micropolar fluid flow and heat transfer past a stretching surface with suction/blowing through a porous media has been studied at by Eldabe *et al.* [4]. Sajjad et al. [5] investigated hydromagnetic micropolar fluid flow between two parallel plates, the lower plate is stretching. Barika and Dashb [6] studied the peristaltic motion of incompressible micropolar fluid through a porous medium in a two-dimensional symmetric channel. Vimala and Omega [7] analyzed the steady laminar two-dimensional flow of a micropolar fluid through a porous channel with variable permeability. The Magnetohyderodynamic (MHD) viscous flow of micropolar fluid over a shrinking sheet has been solved numerically by Shafique [8].

Latterly, heat transfer in micropolar fluids was also discussed by Eringen [9]. Swapna *et al.* [10] developed a mathematical model to study the effects of thermal radiation and uniform transverse magnetic field on the mixed convective flow of a micropolar fluid near the stagnation point on a heated vertical permeable plate, in the presence of constant suction velocity.

Ahmad *et al.* [11] investigated unsteady blood flow having micropolar fluid properties with heat source through parallel plates channel under the influence of a uniform transverse magnetic field. Khilap and Manoj [12] analyzed the fluid flow and heat transfer characteristics occurring during the melting process due to a stretching surface in a micropolar fluid with thermal radiation. Odelu and Kumar [13] investigated incompressible two-dimensional heat and mass transfer of an electrically conducting micropolar fluid flow in a porous medium between two parallel plates with chemical reaction with Hall and ion slip effects. Alam [14] examined the unsteady two dimensional hydromagnetic forced convective heat transfer flow of a viscous incompressible micropolar fluid along a permeable wedge with convective surface boundary condition. Hussain et al. [15] investigated MHD boundary layer flow and heat transfer for micropolar fluids over a shrinking sheet. El-Hakiem *et al*. [16] examined the effects of magnetic field and double dispersion on mixed convection heat and mass transfer in non-darcy porous medium.

In this work, we considered micropolar fluids flow over a shrinking porous surface in the presence of magnetic field and thermal radiation to extend the work of Santosh and Kumar [17]. The numerical solution of the problem has been sought to examine the nature of fluid flow, microrotation and heat transfer. The flow with shrinking surface and the effect of micropolar material constants and radiative heat source put new dimension to the problem.

2. MATHEMATICAL MODEL

Consider micropolar fluid flow towards the stagnation point on a porous stretching surface**.** The fluid is incompressible and electrically conducting. The flow is steady and two-dimensional. The magnetic field of strength H_o is perpendicular to the surface that stretches or shrinks along x-axis. The horizontal component of velocity varies proportional to a specified distance x. The velocity of flow in the region exterior to the boundary layer is $U=cx$. The surface temperature is T. The temperature in the region exterior to the boundary layer is T_∞ . The body couple is absent. Velocity vector is $\underline{V}=V(u,v)$ and spin vector of microrotation is $\underline{\omega}=\omega(0,0,\omega_3)$

Under the above assumptions the equations governing the problems are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$(\mu+k)\frac{\partial^2 u}{\partial y^2}+k\frac{\partial \omega_3}{\partial y}-\sigma\mu_0^2H_0^2u + \rho U\frac{dU}{dx}=\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}) \quad (2)$$

$$\gamma(\frac{\partial^2\omega_3}{\partial y^2})-\kappa(\frac{\partial u}{\partial y}+2\omega_3) = \rho j\left(u\frac{\partial \omega_3}{\partial x}+v\frac{\partial \omega_3}{\partial y}\right) \quad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}(\frac{\partial u}{\partial y})^2 -\frac{16\alpha}{3\beta\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\sigma\mu_0^2H_0^2}{K\rho C_p}u^2 \quad (4)$$

Where ρ is density, σ is the electrical conductivity, K is the thermal conductivity, C_p is the specific heat capacity at constant pressure, μ is dynamic viscosity, k and γ are additional viscosity coefficients for micropolar fluid and j is micro inertia , α is Stefan Boltzmann constant, β is Roseland mean absorption coefficient and μ_0 is magnetic permeability.

The boundary conditions are:

$$\omega_3(x,0)=0,\ u(x,0)=cx,\quad v(x,0)=\pm v_0,\quad T(x,0)=T_w$$

$$\omega_3(x,\infty)=0,\ u(x,\infty)=ax,\quad v(x,\infty)=-ay, T(x,\infty)=T_\infty \quad (5)$$

Using the following similarity transformations:

The velocity components are described in terms of the stream function Ψ (x, y):

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$$

$$\psi(x,y)=x\sqrt{a\upsilon}\,f(\eta),\ \eta=y\sqrt{\frac{a}{\upsilon}}$$

$$u=xaf',\quad v=-\sqrt{\upsilon a}\,f,$$

$$\omega_3=a\sqrt{\frac{a}{\upsilon}}xL(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equation (2) to equation (4), we get:

$$(1+d_1)f'''+d_1L'-Hf'+1 = f'^2-ff'' \quad (6)$$

$$d_3L''+2d_1d_2L-d_1d_2f'' = f'L-fL' \quad (7)$$

$$(4+3R_n)\theta'' +3R_n\Pr(f\theta'+E_cf''^2+Hf'^2)=0 \quad (8)$$

The boundary conditions (5) become:

$$f'(0)=\lambda,\ f(0)=A,\ L(0)=0,\ \theta(0)=1,\quad f'(\infty)=1,\ L(\infty)=0,\ \theta(\infty)=0. \quad (9)$$

Where $A=\pm\frac{v_0}{\sqrt{a\upsilon}}$, $P_r=\frac{\mu C_p}{k}$

$$E_c=\frac{U^2}{c_p(T_w-T_\infty)},\ H=\frac{\mu_0^2H_0^2\sigma}{\rho a}$$

$$R_n=\frac{\beta k}{4\alpha T_\infty^3},\ \lambda=\frac{c}{a}$$

and the dimensionless material constants are:

$$d_1=\frac{k}{\mu}, d_2=\frac{\mu}{\rho ja}, d_3=\frac{\gamma}{\rho j\upsilon}$$

3. RESULTS AND DISCUSSION

By using Mathematica, computations have been carried out for the non-linear model of this study as described finally in equations (5) to (8), in order to observe the flow dynamics. The impacts of the effective physical parameters included herein have been observed on temperature distribution, fluid velocity and micromotion. In general, the effects of the parameters namely Hartmann number H, Eckert number E_c and Prandtl number P_r have been supplemented with suction/injection parameter A, stretching/shrinking parameter λ, radiation parameter R_n, and the material parameters d_1 ,d_2, d_3. In specific, the material parameters and the microrotation function L corresponds to the micropolar property of the fluid and its motion. The values of the material parameters d_1, d_2, d_3 have been chosen arbitrarily and fixed values of these parameters are d_1=0.5, d_2=1.5, d_2= 2.0. The results have been presented in graphical form for the representative values of the parameters of interest.

It is noticed that the magnetic force impedes the fluid flow, whether the surface is normal or it is shrinking. The fluid velocity component f' reduces in magnitude with increase in the values of Hartmann number H as depicted in fig.1 and fig.2 (shrinking surface). However, the micropolar parameter d_1 causes an increasing effect on f' as shown in fig.3. The fluid velocity is larger in magnitude for suction than for injection as demonstrated in fig.4.

Fig.5 shows that micromotion of the fluid particles increases with increase in the values of material parameter d_1. The fig.6 and fig.7 respectively indicate that the magnetic force and suction/ injection reduces the micromotion L near the surface and then enhances it in rest of the region beyond the surface. Fig.8 and fig.9 respectively demonstrate that the temperature distribution decreases with increase in the values of Prandtl number P_r and radiation parameter R_n. But the temperature function $\theta(\eta)$ increases with increase in Eckert number E_c as shown in fig.10.

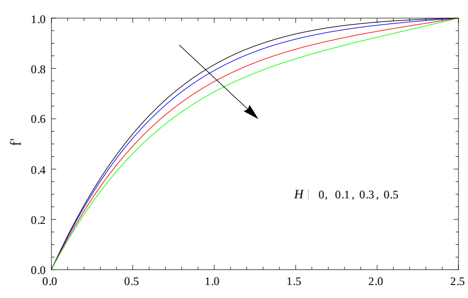

Fig.1: The plot for curves of f' under the effect of Hartmann number H when λ=0, A=1, P_r=0.1, E_c=0.1 andR_n=0.1

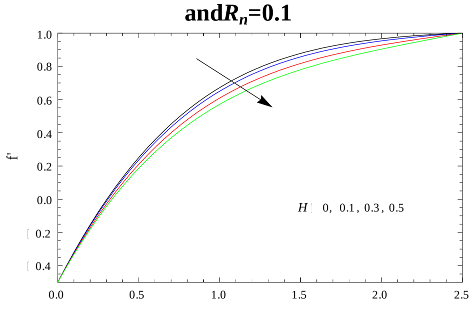

Fig.2: The plot for curves of f' under the effect of Hartmann number H when λ= -0.5, A=1, P_r=0.1, E_c=0.1 and R_n=0.1

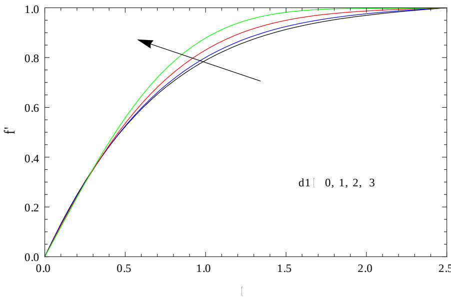

Fig.3: The plot for curves of f' under the effect of micropolar parameter d_1 when λ=0, A=1, P_r=0.1, E_c=0.1 and R_n=0.1

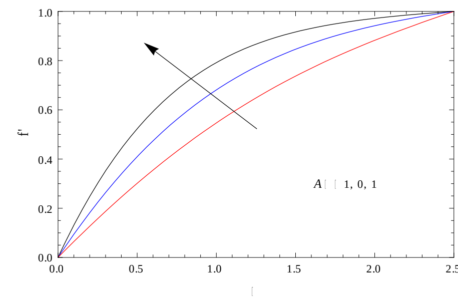

Fig.4: The plot for curves of f' under the effect of suction/ injection parameter A when λ=0, H=0.1, P_r=0.1, E_c=0.1 and R_n=0.1

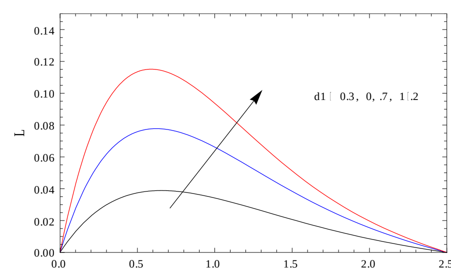

Fig.5: The plot for curves of microrotation L under the effect of d_1 when λ=0, A=1, H=0.1,P_r=0.1, E_c=0.1 and R_n=0.1

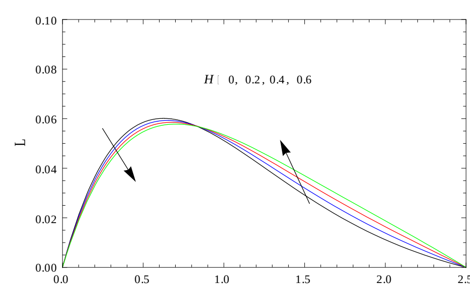

Fig.6: The plot for curves of microrotation L under the effect of Hartmann number H when λ=0, A=1, P_r=0.1, E_c=0.1 and R_n=0.1

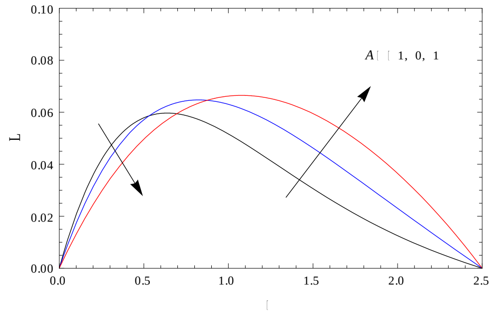

Fig.7: The plot for curves of microrotation *L* under the effect of suction/ injection parameter *A* when λ=0, *H*=0.1, P_r=0.1, E_c=0.1 and R_n=0.1

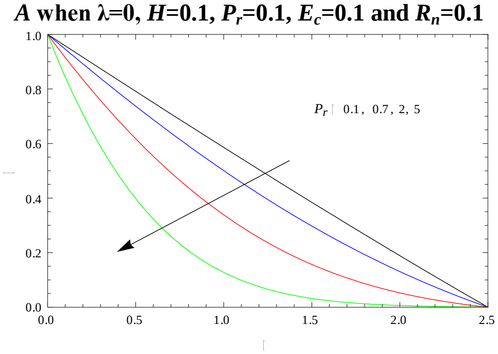

Fig.8: The plot for curves of *θ* under the effect of Prandtl number P_r when λ=0, *A*=1, *H*=0.1, E_c=0.1 and R_n=0.1

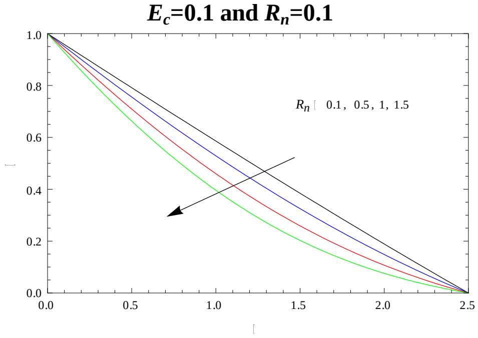

Fig.9: The plot for curves of *θ* under the effect of Radiation parameter R_n when λ=0, *A*=1, P_r=0.1, E_c=0.1 and *H*=0.1

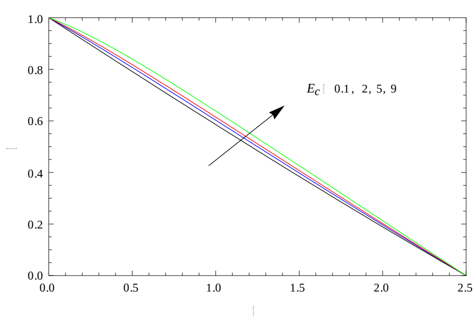

Fig.10: The plot for curves of *θ* under the effect of Eckert number E_c when λ=0, *A*=1, P_r=0.1, *H*=0.1 and R_n=0.1

REFERENCES

[1] Eringen, A. C., Theory of micropolar fluids. Journal of Mathematical Mechanics, 16, 1–18, 1966.

[2] Magyari, E., Pop, L. and Valk, P. P., Stokes' first problem for micropolar fluids. Fluid Dynamics Research, 42, 1–15, 2010.

[3] Takhar, H. S., Bhargava, R., Agrawal, R. S. and Balaji, A.V. S., Finite Element Solution of Micropolar fluid flow and heat transfer between two porous discs. International Journal of Engineering Sciences, 38, 1907-1922, 2000.

[4] Abo-Eldahab, E. M. and El-Aziz, M. A. Flow and heat transfer in micropolar fluid past a stretching surface embedded in a non-darcian porous medium with uniform free stream. Applied Mathematics and Computation, 162, 881-899, 2005 .

[5] Sajjad, H., Muhammad, A. K. and Muhammad, S., Hydromagnetic Flow of Micropolar Fluid between two Horizontal Plates, both the Plates being Stretching Sheets. World Applied Sciences Journal, 28 (3), 1888-1895, 20-34, 2013.

[6] R. N. Barika and G. C. Dashb, Chemical reaction effect on peristaltic motion of micropolar fluid through a porous medium with heat absorption in the presence of magnetic field. Advances in Applied Science Research, 6 (3), 20-34, 2015.

[7] P. Vimala, P. Blessie Omega, Solution of micropolar fluid flow through porous channels – a differential transform approach. Applied Mathematical Sciences, 9 (66), 3291 – 3302, 2015.

[8] Shafique, M., Numerical solution of MHD viscous flow of micropolar fluid over a shrinking sheet using SOR Iterative Procedure . International Journal of Innovation and Scientific Research, 14 (2) Apr. 259-267, 2015.

[9] Eringen, A. C. Theory of thermomicropolar fluids, Journal of Mathematical Analysis and Applications, 38, 480-496, 1972.

[10] Swapna, G., Kumar, L. and Bhardwaj, N., Study of effects of radiation and magnetic field on the mixed convection micropolar fluid flow towards a stagnation point on a heated vertical permeable plate using finite element method. International Journal of MechanicSystems Engineering , 5(1), 1-13, 2015.

[11] F. Ahmad, S. Hussain & A. Ansari, Unsteady MHD blood flow with micropolar fluid characteristics and heat source through parallel plate channel, *J. Appl. Environ. Biol. Sci.* 5(4): 80-86.2015.

[12] Khilap, S., and Manoj K., Effect of thermal radiation on melting heat transfer in stagnation point flow of mhd micropolar fluid towards a stretching surface . International Journal of Advanced Engineering Research and Technology, ICRTIET-2014 Conference Proceeding, 30th -31st August, 2014.

[13] Odelu O., and N. Naresh Kumar, Unsteady heat and mass transfer of chemically reacting micropolar fluid in a porous channel with hall and ion slip currents. Hindawi Publishing Corporation International Scholarly Research Notices Volume 2014, Article ID 646957, 11 pages, 2014.

[14] Alam, M. S., Tarikul I., and M. M. Rehman, Unsteady hydromagnetic forced convective heat transfer flow of a micropolar fluid along a porous wedge with convective surface boundary condition. J. Appl. Environ. Biol. Sci., 5(8S) 43-51, 2015.

[15] Hussain, S., Bagh A., F. Ahmad, MHD boundary layer flow and heat transfer for micropolar fluids over a shrinking sheet, *J. Appl. Environ. Biol. Sci.* 5(5): 330-338, 2015.

[16] El-Hakiem, M. A., F. Ahmad, S. Hussain, Effect of magnetic field and double dispersion on mixed convection heat and mass transfer in non-darcy porous medium, Sci.Int.(Lahore), 27(2), 845-852, 2015.

[17] Santosh, C. and Kumar, P., MHD Stagnation point flow and heat transfer over a permeable surface, Engineering ,5, 50-55, 2013 .