MINIMIZATION OF FLAGPOLE OPTIMIZATION PROBLEM THROUGH DERIVATIVE FREE METHODS

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ABSTRACT: *The purpose of this research is to formulate optimization model flagpole with stress, deflection, and bending shear constraints. Three derivative free methods, namely Nelder-Meed method, Hooke-Jeeve Method and Multidirectional search Methods are used. In this model penalty functions are utilized to remove constraints. In thusly the constrained optimization model is changed into unconstrained model. MATLAB is being used to get results which show the efficiency and success of these methods.*

Keywords: *derivative free methods, penalty function, structural optimization problem, unconstrained optimization*

1. INTRODUCTON

Optimization is very much used in science, engineering, administration, finance and many other fields of life. Mathematical optimization is maximization or minimization of objective function subject to constraints. The main techniques of optimization, namely, derivative based method and derivative free methods (direct search) are being used frequently [1]. Among the direct search methods we focused on Nelder and Mead (NM) method [2, 6], Hooke and Jeeves (HJ) method [4, 5] and Multidirectional Search (MDS) method.

These methods are designed for unconstrained optimization problems. They can be applied to constrained optimization problems by transforming them into unconstrained optimization problems. For the constrained optimization problems the performance of Nelder-Mead method and Hooke-Jeeves methods vary as the nature of the feasible region and the response surface of the objective function changes.

The thought of the multi-directional search calculation originated from the direct search methods for unconstrained optimization. The direct search technique is utilized by the certainty the choice making procedure is focused around exclusively on function esteem data. Distinctive search methods like Hooks and Jeeves and Nelder and Mead utilized before this technique. At last the greater part of the direct search technique are not difficult to utilize, straightforward and simple to make.

We shall assume throughout that an initial estimation of the solution is available. This initial estimate may or may not be feasible. We discuss algorithms that generate a sequence of points. Approximate stationary points of an associated unconstrained function called a penalty function. The original constrained problem is transformed into unconstrained optimization problems by using penalty function [3, 8-10].

2. MATERIAL AND METHODS

2.1 Nelder- Meed Simplex Method

Reflection: Reflection take place when $x^G \geq x^R > x^B$.

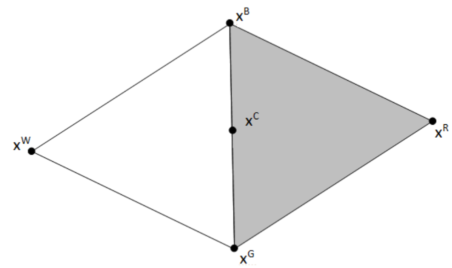

Fig 1: Reflection for Nelder- Meed Simplex Method

Expansion: Expansion take place when $x^G \geq x^B > x^E$.

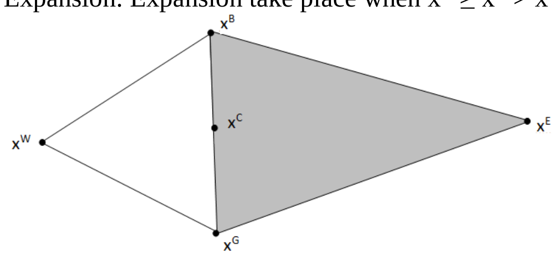

Fig 2: Expansion for Nelder- Meed Simplex Method

Outer contraction: Outside contraction take place when $x^W \geq x^R > x^G$.

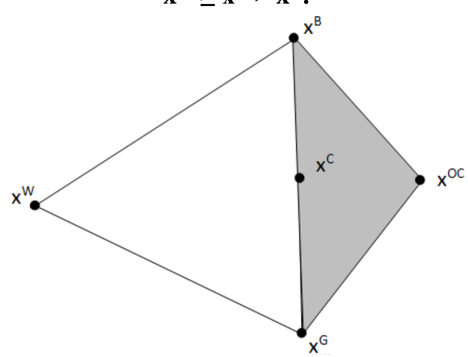

Fig 3: Outer contraction for Nelder- Meed Simplex Method

Inner contraction: Inside contraction takes place when $x^R \geq x^W$.

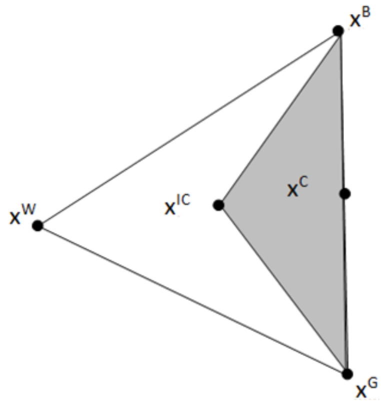

Fig 4: Inner contraction for Nelder- Meed Simplex Method

Shrink: At the point when all the function values are greater than the function value best case scenario point then shrink exist

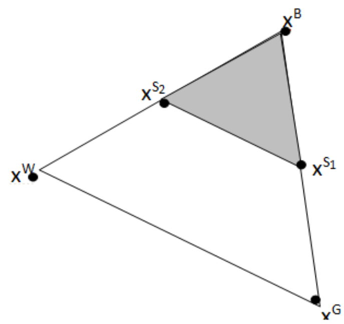

Fig 5: Shrink for Nelder- Meed Simplex Method

2.2 Hook- Jeeves Method

This method works with two types of moves [4].

- Exploratory move
- Pattern move

Exploratory move: The move which is performed at the current base point x_c to investigate the conduct of the objective function in the area of x_e is called an exploratory move [5].

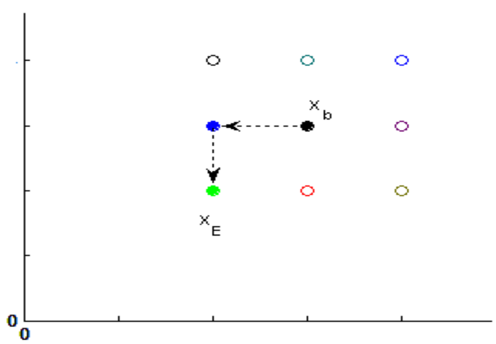

Fig 6: Exploratory move for Hook- Jeeves Method

Pattern move: A fruitful exploratory move gives two focuses. One of these is beginning base point x_b and the other point is x.

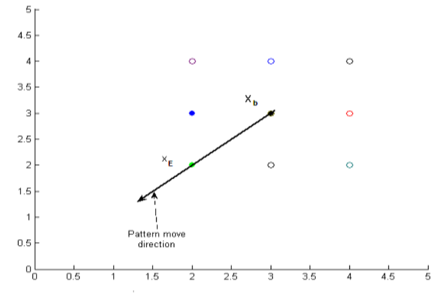

Fig 7: Pattern move for Hook- Jeeves Method

2.3 Multi-directional search Method

Any iteration in multi-directional search calculation we take N + 1 point for N variables. Which characterize in deteriorate simplex [7]. The system utilizes the accompanying operations:-

- reflection
- expansion
- inner Contraction

Reflection: In the wake of reflecting the first simplex through the best point give another simplex.

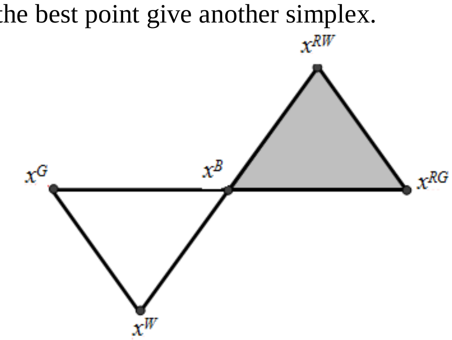

Fig 8: Reflection for Multi-directional search Method

Expansion: We expand the reflected simplex by doing the length of two times of each edge along reflected simplex for this one of the reflected point < best point.

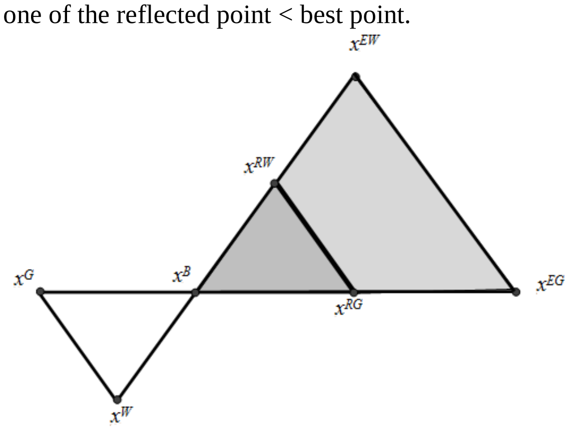

Fig 9: Expansion for Multi-directional search Method

Inner contraction: If the function value of the reflect point ≥ function value of best point then inner contraction has done at the best point by doing half the length of each edging.

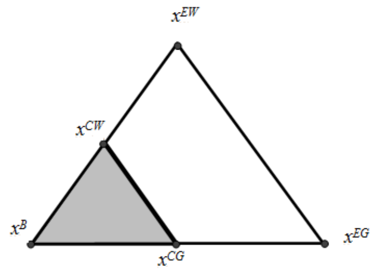

Fig 10: Inner contraction for Multi-directional search Method

3. STRUCTURAL OPTIMIZATION PROBLEMS DESIGN PROBLEM

Minimize the mass of a standard 10 m tubular flagpole to withstand wind gusts of 350 mile/h. The flagpole will be made of structural steel. Use a factor of safety of 2.5 for structural design. The deflection of the top of flagpole should not exceed 5 cm.

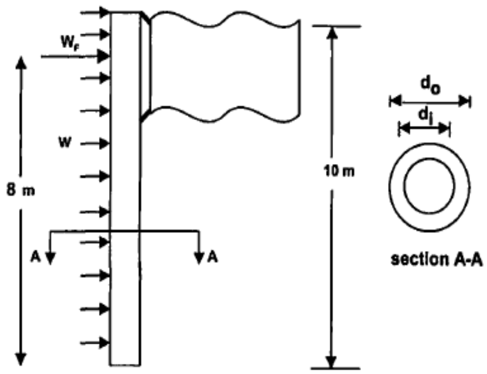

Fig 11: Model of flagpole

Design Parameters:

E (modulus of elasticity): 200E + 09 Pa
σ_{all} (allowable normal stress): 250E + 06 Pa
τ_{all} (allowable shear stress): 145E + 06 Pa
γ (material density): 7860 kg / m^3
FS (factor of safety): 2.5
g(gravitational acceleration) : 9.8 m/s^2

Aerodynamic Calculation:

ρ (Standard air density): 1.225 kg / m^3
C_d (drag coefficient of cylinder): 1.0
W_F (flag wind load at 8 m): 5000 N
V_w (wind speed): 350 mph or 156.46 m/s

Geometric Parameters:

L_p: Location of flag wind load: 8 m
L: Length of pole: 10 m
δ_{all} : permitted deflection: 5 m

Design Variable:

d_o: outer diameter (x_1)
d_i: inner diameter (x_2)

Geometric Relations:

A = area of cross section
$= 0.25\,\pi\,(d_o^2 - d_i^2)$
$= 0.785(d_o^2 - d_i^2)$

I = diametrical moment of inertia
$= \pi\,(d_o^4 - d_i^4)/\,64$
$= 0.049\,(d_o^4 - d_i^4)$

Q/ t = first moment of area above the neutral axis divided by thickness
$= (d_o^2 + d_o\ d_i + d_i^2)\,/\,6$

Objective Function:

Minimize $z = f(x_1, x_2) = L\,A\,\gamma\,g$

Subject to

$g_1(x_1, x_2)$: $\sigma_{bend} + \sigma_{weight} \le \sigma_{all}/\,FS$

g_1 shows the normal stress

$g_2(x_1, x_2)$: $\tau \le \tau_{all}\,/\,FS$

g_2 shows the shear stress

$g_3(x_1, x_2)$: $\delta_w + \delta_F \le \delta_{all}$

g_3 shows the deflection constraints

$g_4(x_1, x_2)$: $d_o - d_i \ge 0.001$

g_4 is the geometric constraints

$2cm \le d_o \le 100$ cm
$2cm \le d_i \le 100$ cm

Calculation:

$Z = f(x_1, x_2) = L\,A\,\gamma\,g$
$= 10 \times 0.785(d_o^2 - d_i^2) \times 7860 \times 9.81$
$= 605286.810\,(d_o^2 - d_i^2)$

$$\sigma_{bend} = \frac{0.5\,(M_w + M_F)d_o}{I} \qquad (1)$$

$M_w = 0.5\,F_D\,L^2$
$= 0.5\,(0.5\rho\,v_w^2\,C_d\,d_o)$
$= 0.5(0.5 \times 1.225 \times 156.46^2 \times 1.0\,d_o\,(10)^2$
$= 749691.780\,d_o$

$M_F = w_F\,L_P$
$= 5000 \times 8 = 40000$

Put the values in (1)

$$\sigma_{bend} = \frac{0.5\,(749691.780\,do + 40000)\,do}{0.049(d_o^4 - d_i^4)}$$
$$= \frac{10.204\,(749691.780\,do + 40000)\,do}{d_o^4 - d_i^4}$$

$\sigma_{weight} = \gamma\,g\,L$
$= 7860 \times 9.81 \times 10 = 771066$

$$\tau = \frac{SQ}{It} \qquad (2)$$

$S = W_F + F_D L$
$S = 5000 + (14993.836 d_o) \times 10$
$S = 5000 + 149938.360 d_o$

Put the values in (3.2)

$$\tau = \frac{(5000+149938.360 d_o)\,(d_0^2 + d_o d_i + d_i^2)}{0.294\,(d_o^4 - d_i^4)}$$

$$\delta_w = \frac{F_D\,L^4}{8\,E\,I} \qquad (3)$$

where δ_w is the deflection at the top due to uniform wind load, put the values in (3)

$$\delta_w = \frac{14993.8360\,d_o 10^4}{8 \times 200 \times 0.049(d_o^4 - d_i^4)}$$
$$= \frac{191247.908 d_0}{d_o^4 - d_i^4}$$

$$\delta_F = \frac{2\,w_F\,L^3 - w_F\,L^2\,L_p}{E\,I} \qquad (4)$$

Where δ_F is the deflection at the top due to the flag wind load at L_p Put the values in (4)

$$\delta_F = \frac{2 \times 5000 \times 10^3 - 5000 \times 10^2 \times 8}{200 \times 0.049\,(d_o^4 - d_i^4)}$$
$$= \frac{612244.898}{d_o^4 - d_i^4}$$

Now constraints become

g_1: $\sigma_{bend} + \sigma_{weight} \le \sigma_{all}/\,FS$

$$\frac{10.204\,(749691.780\,do + 40000)\,do}{d_o^4 - d_i^4} + 771066 \le {}^{250}/_{2.5}$$

The first constraint handles the Normal stresses, and after calculation it becomes

$$770966\, d_o^4 + 7649654.923\, d_o^2 + 408160 d_o - 770966 d_i^4 \leq 0$$

$g_2 : \tau \leq \tau_{all} / FS$

$$\frac{(5000+149938.360 d_o)(d_o^2+d_o d_i+d_i^2)}{0.294\,(d_o^4 - d_i^4)} \leq \frac{145}{2.5}$$

g_2 handles the shear stress in the flagpole and after calculation result is

$(5000+149938.360d_o)(d_o^2+d_od_i+d_i^2) - 17.052\,(d_o^4 - d_i^4) \leq 0$

g_3: $\delta_w + \delta_F \leq \delta_{all}$

$$\frac{191247.908 d_0}{d_o^4 - d_i^4} + \frac{612244.898}{d_o^4 - d_i^4} \leq 0.05$$

g_3 is the deflection constraint and after calculation it becomes

$$191247.908\, d_o - 0.05\,(d_o^4 - d_i^4) + 612244.898 \leq 0$$

g_4 is geometric constraint

g_4: $d_o - d_i \geq 0.001$

$$d_o - d_i - 0.001 \geq 0$$

Formulation of Model: Our final unconstrained model becomes

$Z = 605286.810\,(d_o^2 - d_i^2) + 1000\,[\max\,(770966\,d_o^4 + 7649654.923\,d_o^2 + 408160 d_o - 770966 d_i^4\,),\ ((5000 + 149938.360\, d_o)\,(d_o^2 + d_o d_i + d_i^2) - 17.052\,(d_o^4 - d_i^4))\,,\ (191247.908\, d_o - 0.05\,(d_o^4 - d_i^4) + 612244.898\,)\,,\ (d_o - d_i - 0.001\,)]$

4. RESULTS AND DISCUSSION

The result which obtained from flagpole by applying NM method is that, by taking the initial guess from -1 to 10 .many points are being checked between this ranges. We get many solutions but result does not show consistency and final solution which is feasible as it satisfy all constraints. The function value is 1.948×10^8 at the point (-0.3019, 25.0359).

Table-1: Result of flagpole by applying NM Method

Initial guess	Function value	Final point	Function value	Iteration Count
(-0.29, 9.0060)	5.3623×10^9	(-0.3019 25.0359)	1.948×10^8	73

The result which obtained from flagpole by applying HJ method is that, by taking the initial guess from -1 to 10.Many points are being checked between this ranges. We get many solutions but result shows no consistency. So final solution which is feasible as it satisfy all constraints. The function value is 2.0305×10^8 at the points (-0.3017 24.7326).

Table-2: Result of flagpole by applying HJ Method

Initial guess	Function value	Final point	Function value	Iteration Count
(-0.29, 9.006)	5.3623×10^9	(-0.3017, 24.7326)	2.0305×10^8	23

The result which obtained from flagpole by applying MDS method is that, by taking the initial guess from -1 to 10. Many points are being checked between these ranges. We get many solutions but result is inconsistent and final solution which is feasible as it satisfies all constraints. The function value is 2.030×10^8 at the points (-0.3017, 24.7326)

Table-3: Result of flagpole by applying MDS Method

Initial guess	Function value	Final point	Function value	Iteration Count
(-0.29, 9.0060)	5.3623×10^9	(-0.3017 , 24.7320)	2.030×10^8	305

Nelder and Mead Search Method is better than the other two methods because the function value is smaller than the function value of other two methods. But the number of iterations of Hooks and Jeeves Methods is smaller than number of iterations of the other two methods.

5. CONCLUSION

We have applied MDS method, HJ method and NM method to solve flagpole optimization problem and two bar truss. These methods are also implemented in MATLAB on formulated problems by many times by choosing different step size and initial guess. We conclude that NM method gives optimum solution but its convergence is very far. While Hooks and Jevees gives optimum solution in less no. of iterations other two methods.

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